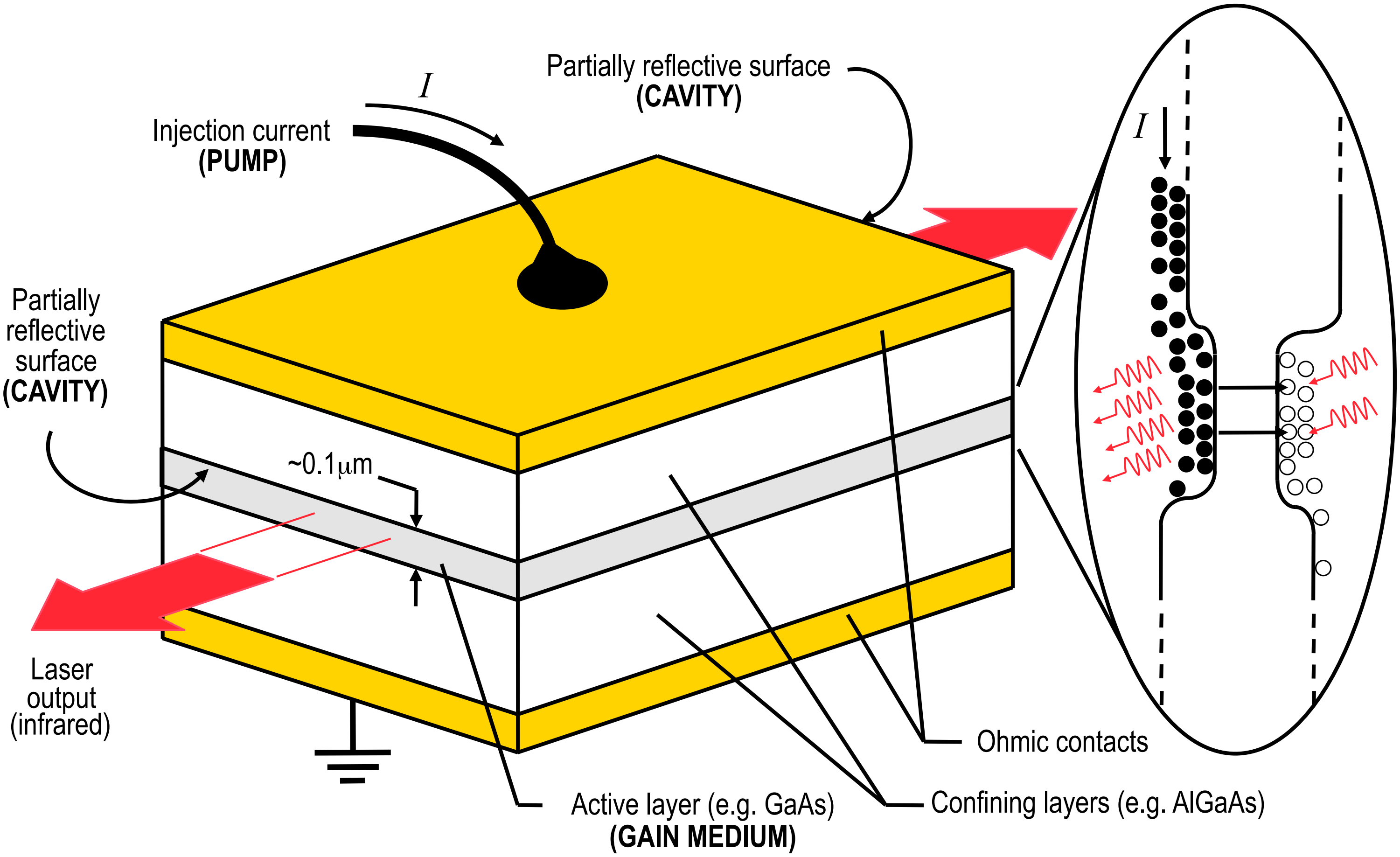
Semiconductor Fundamentals and Optical Gain
ELEC 413

based on Yariv - Chapter 15

Dr. Lukas Chrostowski

Simplified DH Semiconductor Laser



Semiconductor Fundamentals

- Semiconductor crystal structure
- Energy band model
- Charge carriers: electrons and holes
- Concept of effective mass
- Density of states function
- E-k diagrams
- Direct and indirect gap materials

- Fermi-Dirac distribution function
- Intrinsic and extrinsic semiconductor
- Equilibrium carrier distributions
- Quasi-Fermi levels
- Alloying to alter gap energy
- Heavy doping and bandtailing
- PN junction diode and heterojunction

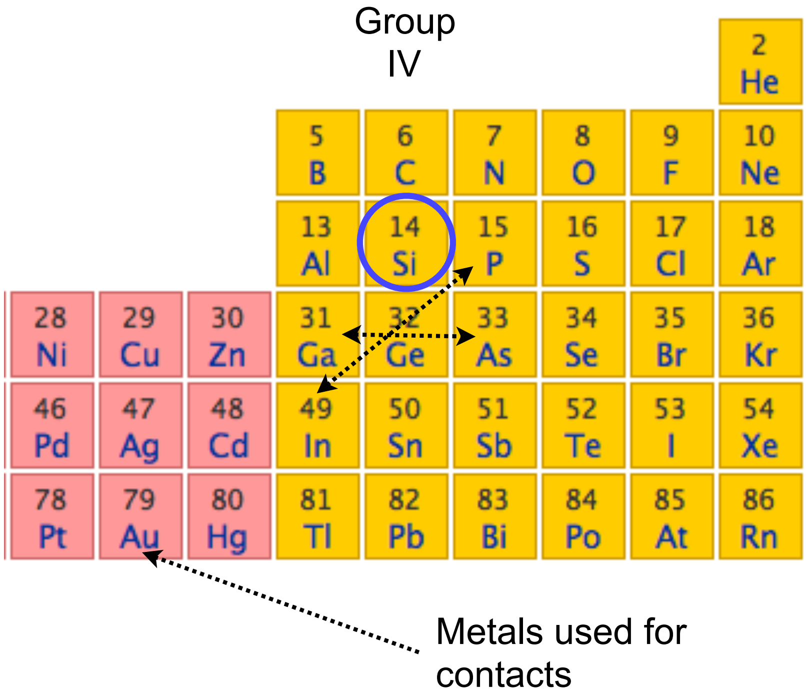
What are Semiconductor Materials ?

- electrical conductivities
  - intermediate between metals (conductor) and insulators.
  - can be varied over orders of magnitude by changes in impurity concentration, optical excitation and temperature.
- variability of electrical properties
  - makes semiconductors useful for constructing devices.
- Semiconductor materials
  - are composed of elements located in column IV of the periodic table, or
  - are a combination of elements in columns an equal distance to either side of column IV.

What are Semiconductor Materials?

- electrical conductivities between metals and insulators.

- can be varied over orders of magnitude by changes in impurity concentration, optical excitation and temperature.



								2 He
			5 B	6 C	7 N	8 O	9 F	10 Ne
			13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn

Metals used for contacts

- Common elements for **optoelectronic devices**:
  - Si, Ge (detectors)
  - Al,Ga (III) As (V) (lasers, detectors, modulators)
  - In (III), P(V) (lasers, detectors)
  - Cd (II), Se (VI) (detectors)
  - B, C, N (dopants)

Comparison of Electronics and Photonics

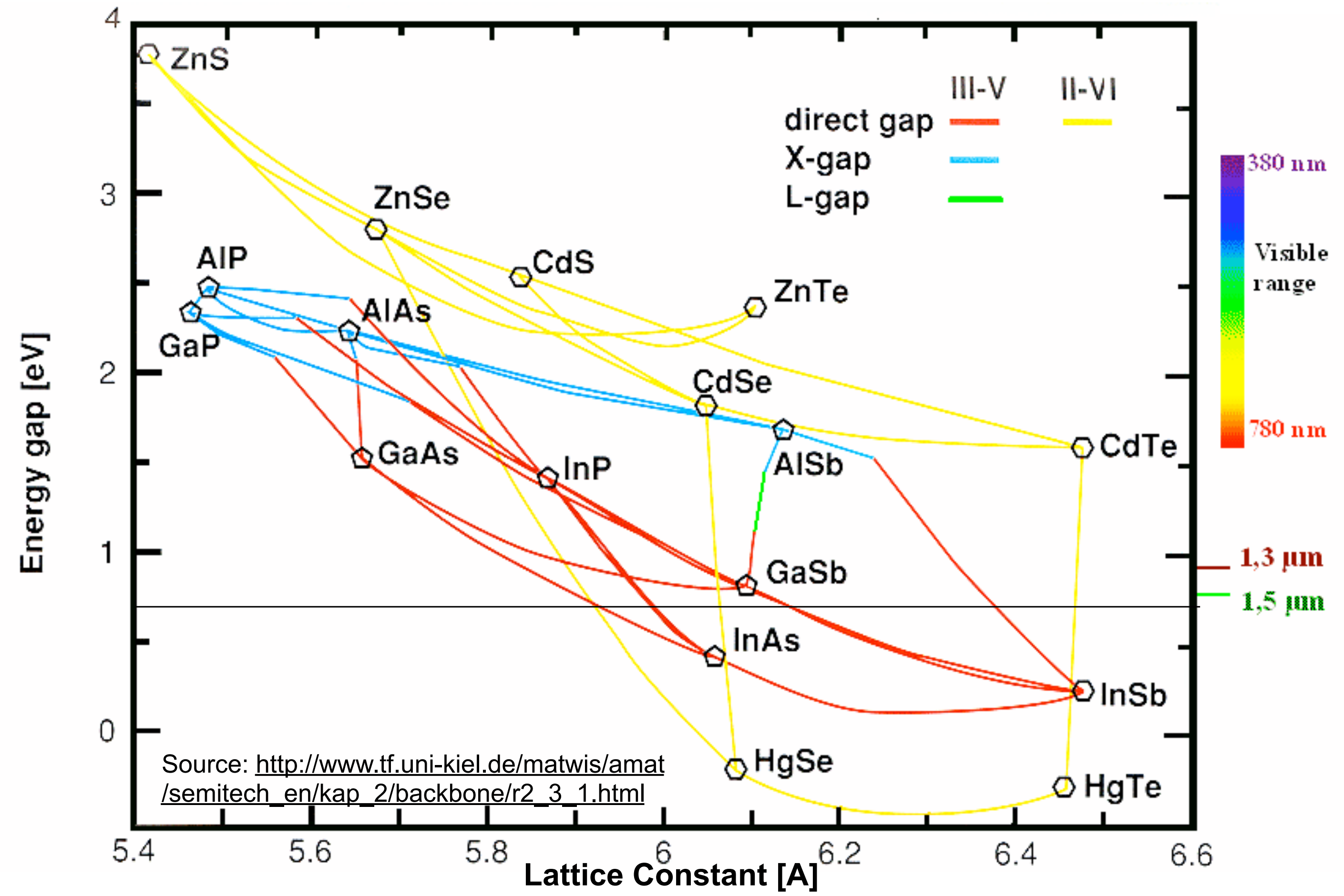
Electronics

- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- $Si/SiO_2/PolySi$
- Si electronics is a processing based technology

Photonics

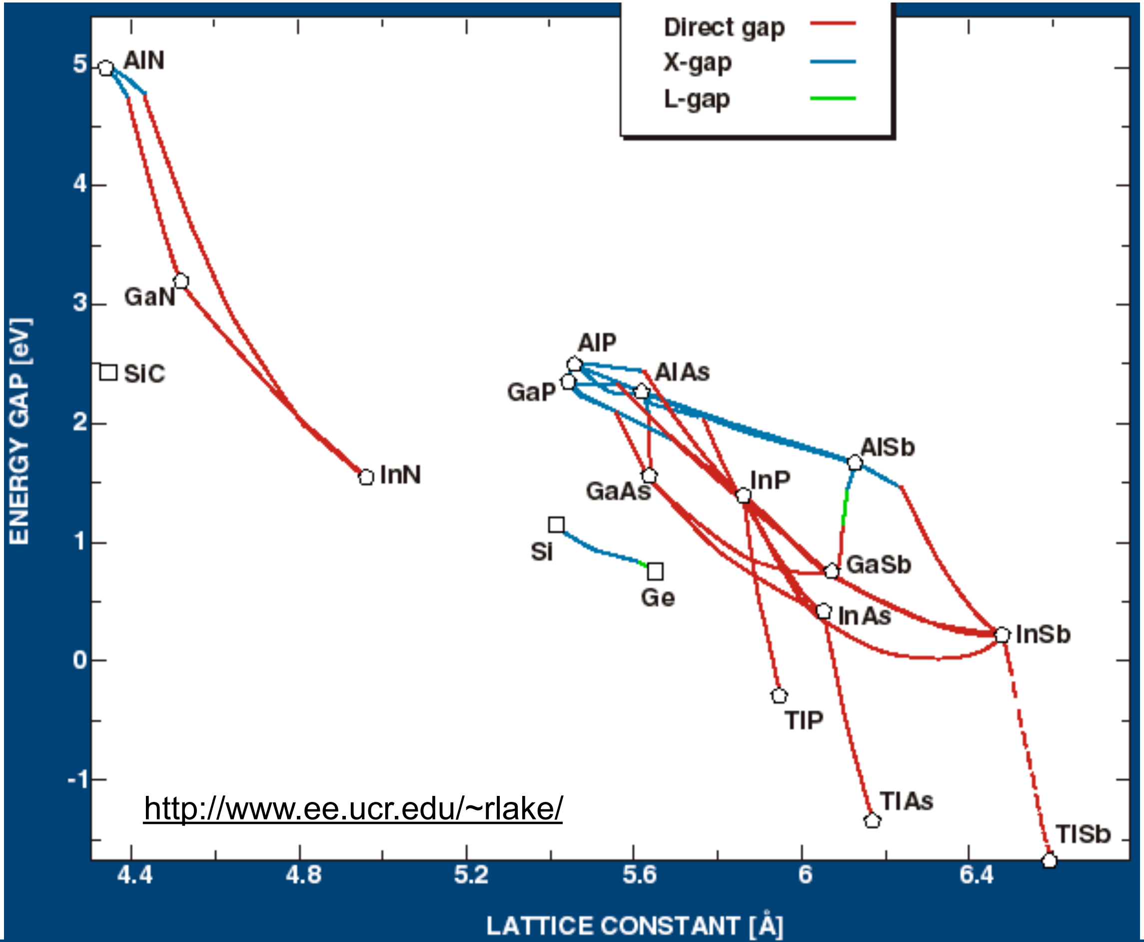
- GaAs, GaP, GaSb, InP, InAs, InSb, AlAs, GaN, AlN
- $GaAs_xP_{1-x}$, $Al_xGa_{1-x}As$, $In_xGa_{1-x}P$, $In_xGa_{1-x}As$, $Al_xGa_{1-x}Sb$, $Al_xGa_{1-x}N$, $Ga_xIn_{1-x}N$--Ternary Alloys
- $In_xGa_{1-x}As_yP_{1-y}$, $Al_xIn_yGa_{1-x-y}As$, $In_xGa_{1-x}As_ySb_{1-y}$, $In_xGa_{1-x}N_yAs_{1-y}$--Quaternary Alloys
- $In_xGa_{1-x}N_yAs_{1-y-z}Sb_z$--Quinary Alloy
- Photonics is a materials based technology

Semiconductor Band Gap vs. Lattice



Source: http://www.tf.uni-kiel.de/matwis/amat/semitech_en/kap_2/backbone/r2_3_1.html

III-V Semiconductor Band Gap vs. Lattice



http://www.ee.ucr.edu/~rlake/

Materials for Lasers & LEDs

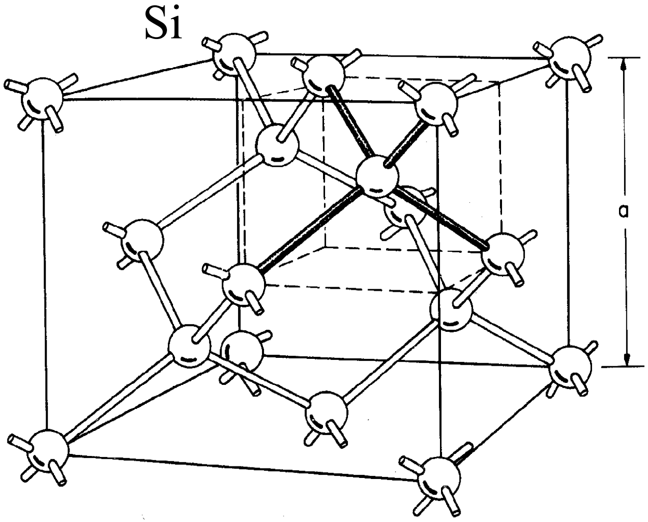
- What material would you use to design a laser or LED for:
  - Visible wavelengths:
  - UV:
  - IR:

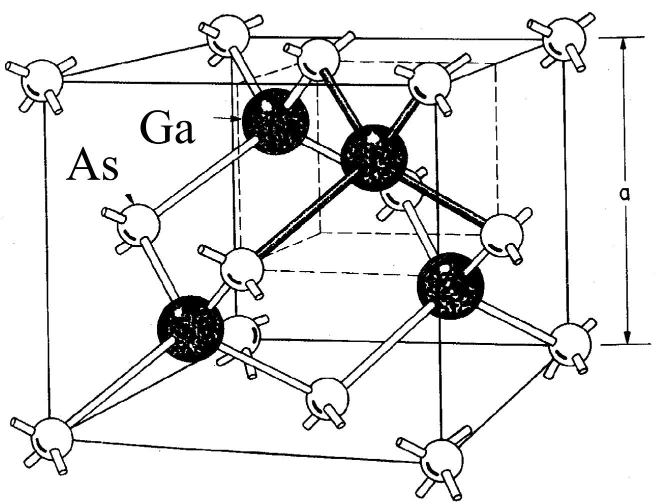
Question

- Show that the wavelength corresponding to a photon energy equal to the bandgap Eg is:

$$\lambda_g[\text{in } \mu m] = \frac{1.24}{E_g[\text{in eV}]}$$

Semiconductor Crystal





Diamond Lattice
Elemental compounds
(Si, Ge)

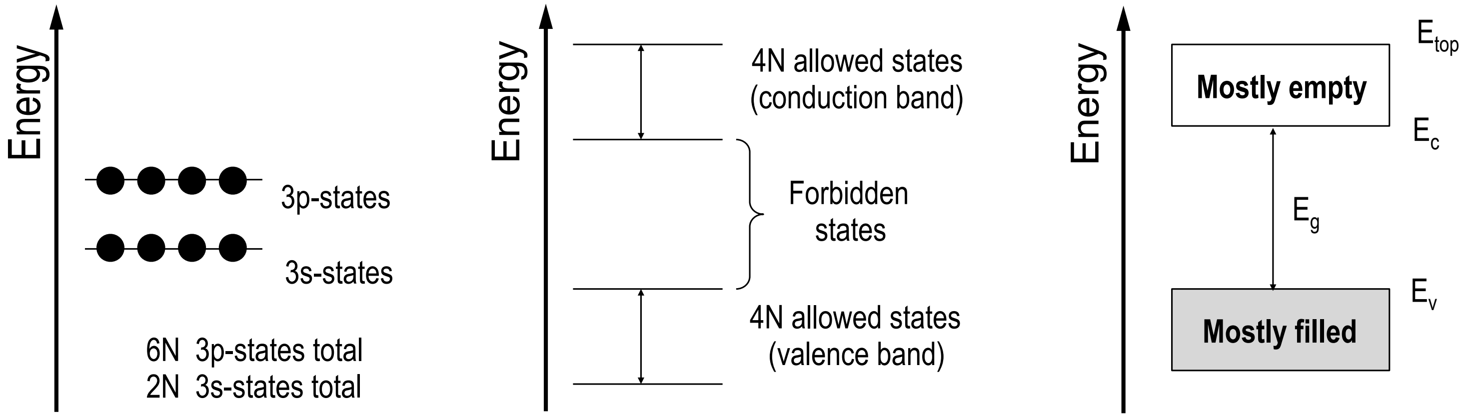
Zincblende Lattice
III-V compounds
(GaAs, InP)

Bonding in Silicon Lattice

- Each Si atom
  - has 4 nearest neighbours, each with 4 electrons in in the outer orbit.
  - shares 4 valence electrons with its 4 nearest neighbours.
- Bonding forces
  - arise from quantum mechanical interaction between shared electrons, known as <u>covalent bonding</u>.
  - Here, each electron pair constitutes a covalent bond.
- <u>At 0K</u>
  - there are no free electrons available to the lattice in the covalent diamond structure.
- Electrons can be thermally or optically excited out of a covalent bond and therefore become free to participate in current conduction.

Development of Si Energy Band Model

N isolated Si atoms → N atoms in Si crystal → Energy band model


Energy

3p-states

3s-states

6N 3p-states total
2N 3s-states total

Energy

4N allowed states (conduction band)

Forbidden states

4N allowed states (valence band)

Energy

E_{top}

Mostly empty

E_c

E_g

E_v

Mostly filled

4N valence electrons

8N allowed states total

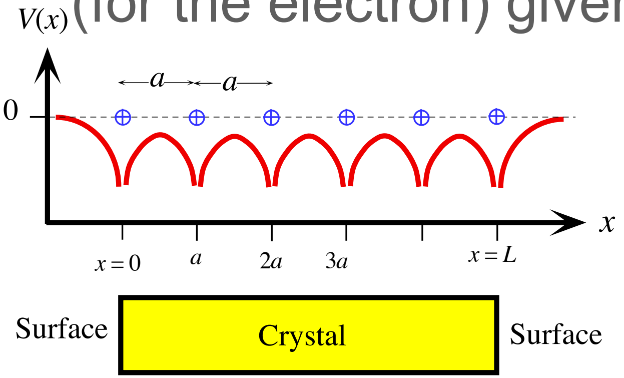
E_g = energy gap

- Pauli exclusion principle
  - each electron must possess a unique spatial wave-function and energy
- Energy levels and occupation determines the conduction properties (e.g. insulator, semiconductor)

Q: Optical emissions?

Schroedinger Equation

- How does an electron behave in a crystal?
- From quantum mechanics, an electron in an infinite potential energy well has quantized energy: $E_n = \frac{\hbar^2 k_n^2}{2m_e}$
- For a semiconductor, solve the Schroedinger equation (for the electron) given an atomic potential V(x):



PE of the electron, *V*(*x*), inside the crystal is periodic with a period *a*.

$$\frac{d^2\psi}{dx^2} + \frac{2m_e}{\hbar^2}[E - V(x)]\psi = 0$$

The electron potential energy (*PE*), *V*(*x*), inside the crystal is periodic with the same periodicity as that of the crystal, *a*. Far away outside the crystal, by choice, *V* = 0 (the electron is free and *PE* = 0).

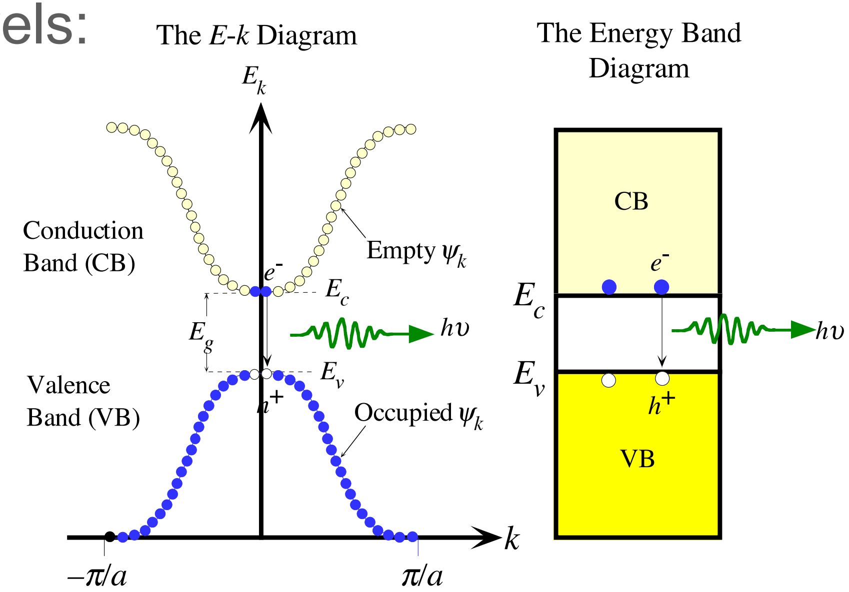
Band diagram

- Solution yields many wavefunctions: $\psi_k(x) = U_k(x)e^{jkx}$
- with different energy levels:

$$E_n \approx \frac{\hbar^2 k_n^2}{2m_e}$$

- with a crystal momentum

$$\hbar k$$



The *E*-*k* diagram of a direct bandgap semiconductor such as GaAs. The *E*-*k* curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the *E*-*k* relationship as a continuous curve. In the energy range E_v to E_c there are no points ($\psi_k(x)$ solutions).

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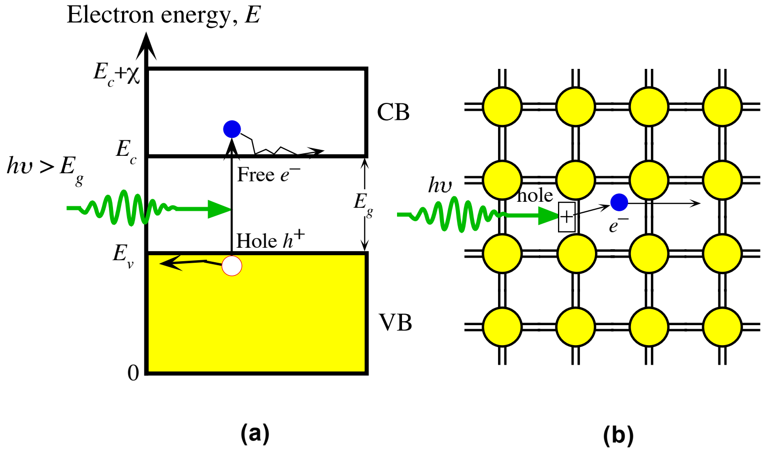
Charge Carriers in Semiconductors

- As we raise the temperature of a semiconductor, some electrons in the valence band receive sufficient thermal energy to be excited across the band gap to the conduction band. This leaves unoccupied states in an otherwise filled valence band. We refer to these empty states as holes.
- Electrons in the conduction band and holes in the valence bands are charge carriers.
- If a conduction band electron and a valence band hole are created by the excitation of valence band electron to the conduction band, the pair of charge carriers are called an electron-hole pair.



(a) A photon with an energy greater than E_g can excite an electron from the VB to the CB.
(b) Each line between Si-Si atoms is a valence electron in a bond. When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created.

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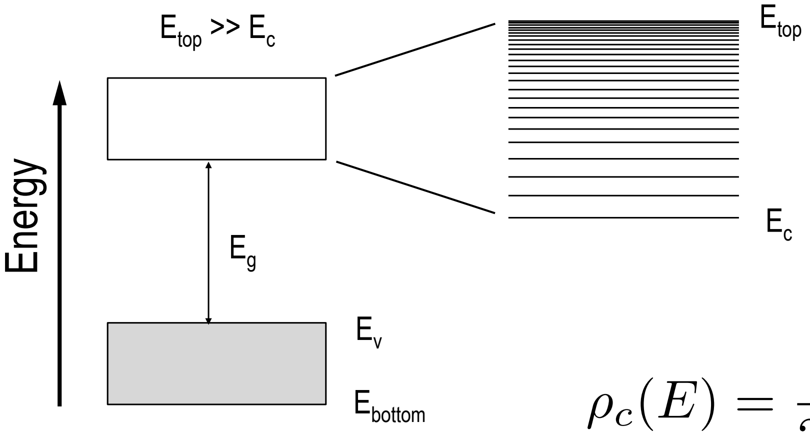
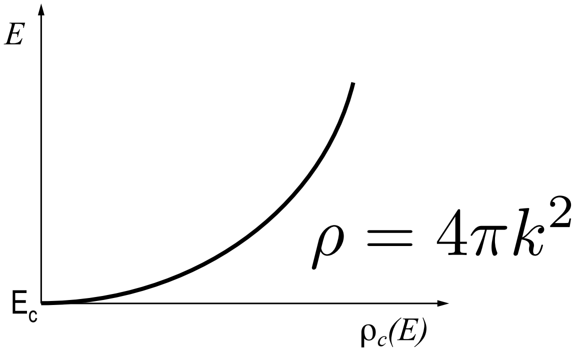
Figure 3.3

Density of States (3D crystal)

- The allowed states in the conduction and valence bands are distributed in energy. This energy distribution of states is described by the density of states $\rho(E)$.
- Plotted with x-axis being the dependant variable (opposite of typical graphs)





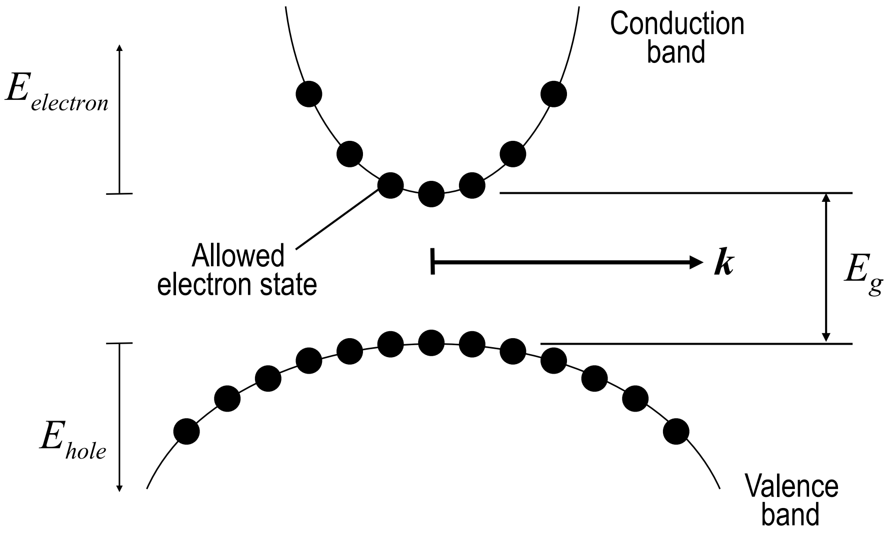
$$\rho = 4\pi k^2$$

$$\rho_c(E) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}(E - E_c)^{1/2}$$

$$\int_{E_1}^{E_2} \rho_c(E)dE$$ = number of conduction band states between E_1 and E_2. (per unit volume)

What is the ideal DOS for a laser?

E-k Diagram for a Direct Gap Semiconductor



$$E_C(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_c}$$

Typical E-k diagram for a direct gap semiconductor with $m_c < m_v$

$$E_V(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_v}$$

When an electron makes a transition (induced or spontaneous) between a conduction band state and one in the valence state, the two states involved must have the same value of k (same crystal momentum). This results from quantum mechanical considerations.

Momentum (k) Conservation

- Transitions between two quantum mechanical states (absorption, emission):
  - Rates proportional to an integral over the crystal volume
  - Involving initial state (2) and final state (1)

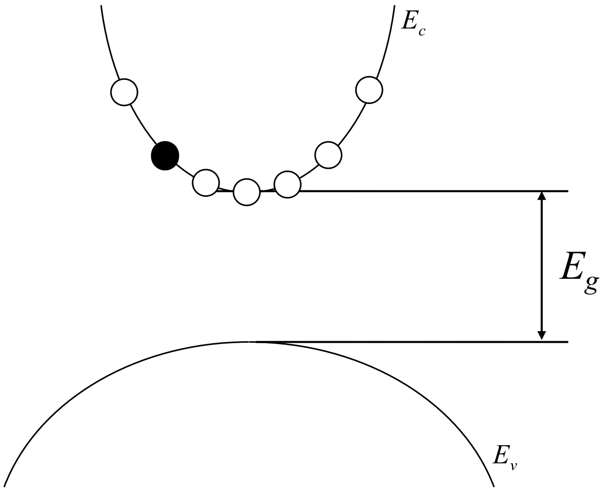
$$\text{rate of transition} \propto \int_V \psi_2(\mathbf{r})\psi_1^*(\mathbf{r})d\mathbf{r}$$

$$\psi_2(\mathbf{r}) = u_{\mathbf{k_2}}(\mathbf{r})e^{i\mathbf{k_2}\mathbf{r}} \text{ (Bloch Theorem)}$$

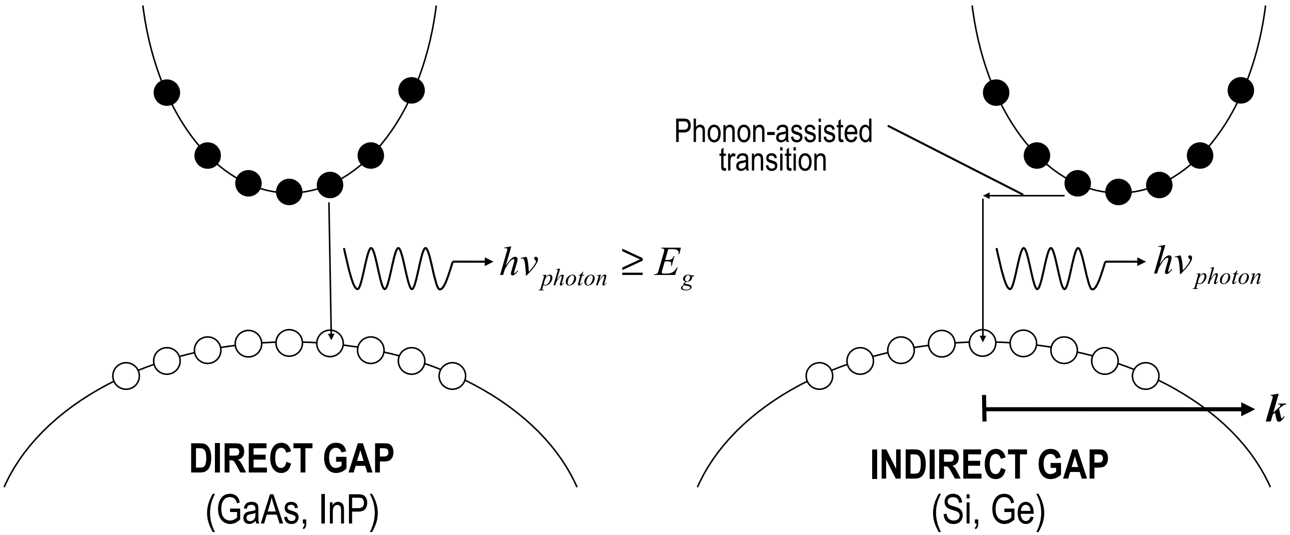
  - For a rate > 0, $\mathbf{k_2} = \mathbf{k_1}$
  - **Thus, transitions much conserve momentum, and appear as vertical arrows on the E-k diagram**

Optical Emission

- Q: Identify and sketch the transition for optical emission
- Q: is the emitted photon wavelength greater or less than the bandgap wavelength?



Direct versus Indirect Gap Semiconductors



Phonon-assisted transition

$h\nu_{photon} \geq E_g$

$h\nu_{photon}$

k

DIRECT GAP
(GaAs, InP)

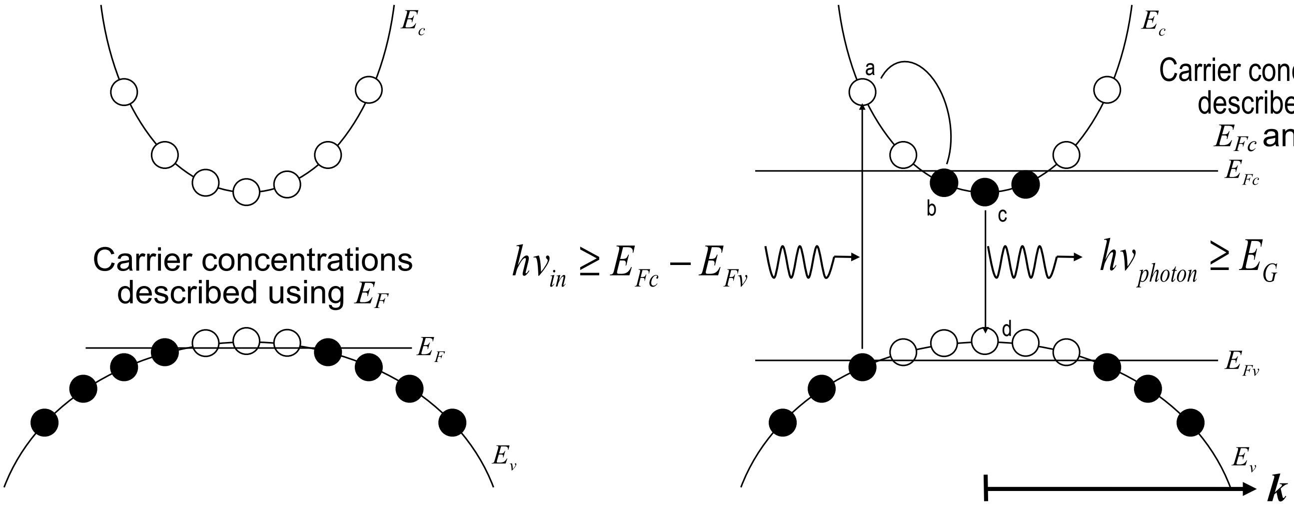
INDIRECT GAP
(Si, Ge)

- When conduction band minimum and valence band maximum occur at the same value of k, the material is said to be *direct gap*. Otherwise it is called *indirect gap*.
- Photon (low momentum, high energy) transitions are vertical lines on a E-k plot.
- Phonon (large momentum, low energy) transitions are horizontal lines on a E-k plot.

Transitions in Direct and Indirect Gap Semiconductors

- In a direct gap semiconductor, conservation of both energy and momentum is met simply by the emission (or absorption) of a photon.
- Photon emission is unlikely in an indirect gap semiconductor: this is because the phonon-assisted emission involves the participation of three bodies (electron, photon, phonon), the probability of their occurrence is quite low.
- Photon absorption is NOT unlikely in an indirect gap semiconductor: this is because absorption can be achieved in a two-step process (photon excitation followed by thermalization).
- Thus, direct gap semiconductors (GaAs, InP) are required to build efficient emitters and amplifiers.
- Indirect or direct gap semiconductor can be used to build detectors, CCDs, solar cells, etc.

Photo-excitation Process



Carrier concentrations described using E_F

Carrier concentrations described using E_{Fc} and E_{Fv}

$h\nu_{in} \geq E_{Fc} - E_{Fv}$

$h\nu_{photon} \geq E_G$

Degenerate p-type material in equilibrium at 0 K

Same degenerate p-type material in non-equilibrium but steady-state conditions at 0 K semiconductor is illuminated with photons of energy $h\nu_{in}$

a→b transitions = “scattering” or “thermalization”. Occurs over a phonon-electron collision lifetime (~10^{-12}s).

c→d transitions = “recombination”. Occurs over a electron-hole pair recombination lifetime (~10^{-9}s).

Because (scattering lifetime) << (recombination lifetime) ⇒ allows for use of Quasi-Fermi levels.

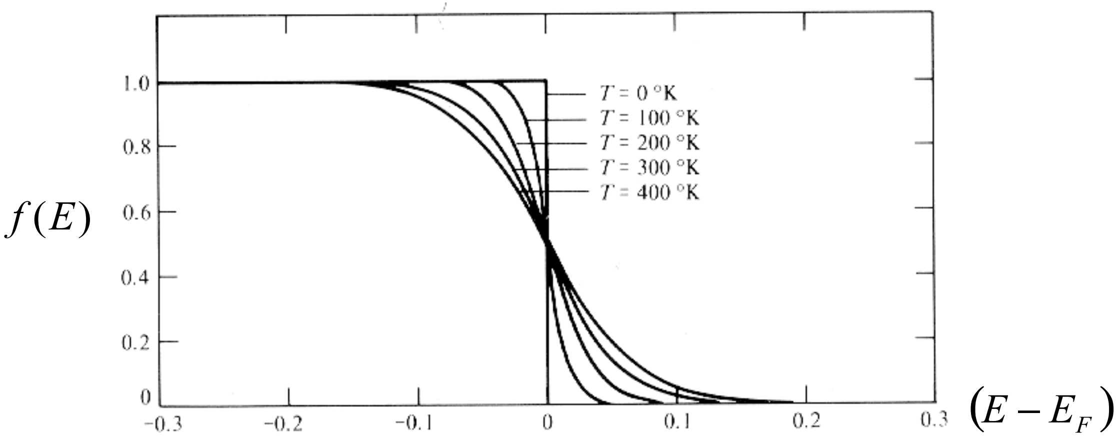
Fermi-Dirac Distribution Function

- In thermal equilibrium, the probability that an electron state of energy E is occupied by an electron is given by the Fermi-Dirac distribution function:

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

k = Boltzmann constant
T = Temperature in Kelvin

- E_F is the Fermi energy; it corresponds to the energy where the occupation probability is ½.

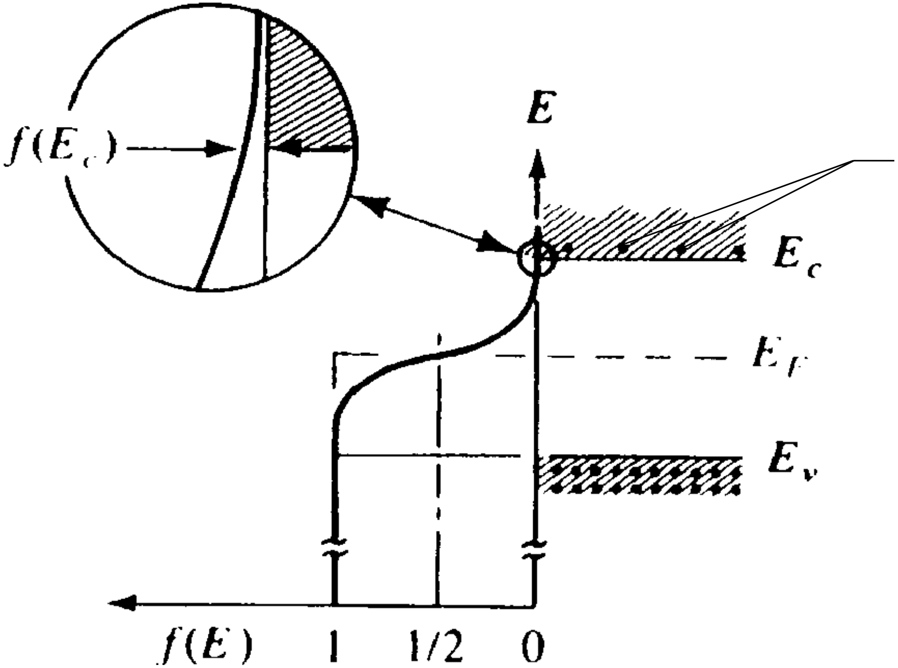


Intrinsic Semiconductor Material

- A perfect semiconductor crystal with no dopant impurities or lattice defects is called an <u>intrinsic</u> semiconductor.
- At 0K, there are no charge carriers. Valence band is completely filled with electrons and conduction band is completely empty.
- Electron-hole pairs are generated thermally as electrons are excited across to the conduction band.
- Since electrons and holes are created in pairs, the conduction band electron concentration n (in #/cm^3) is equal to the valence band hole concentration p (in #/cm^3).
- For an intrinsic semiconductor, one can state: $n = p = n_i$ where n_i is called the intrinsic carrier concentration.

* Intrinsic semiconductors are typically used for active regions (i.e. light generating) in lasers.

Fermi Distribution in an Intrinsic Semiconductor



Thermally excited electrons in the conduction band

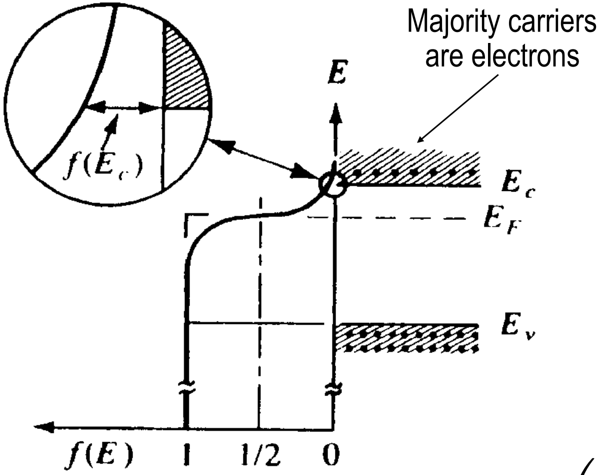
Same number of holes are present in the valence band

For silicon at 300K $\Rightarrow n_i \sim 10^{10}$cm^{-3} while there are 5 x 10^{22} atoms/cm^3.

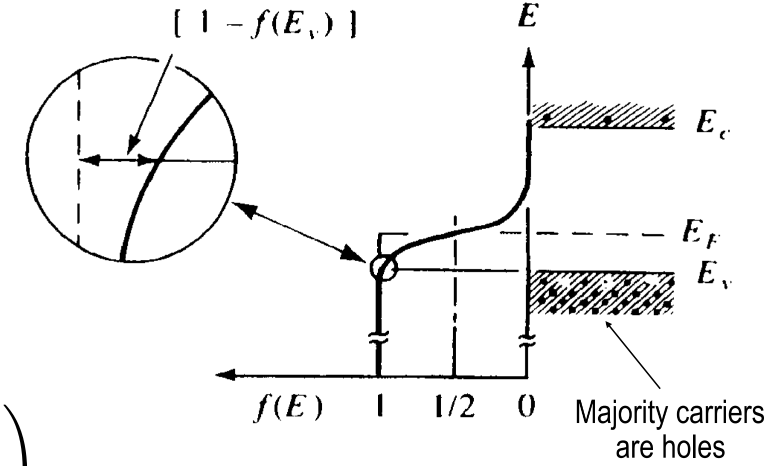
Manipulation of Carrier Numbers - Doping

- Doping = the addition of controlled amounts of specific impurity atoms with the express purpose of increasing either the electron (n) or the hole (p) concentration.
- This is used to vary the semiconductor electrical conductivity by adding free charge carriers (either more electrons or more holes).
- When crystal is doped such that the equilibrium carrier concentrations n_o and p_o are different from the intrinsic carrier concentration n_i, the material is said to be <u>extrinsic</u>.
- N-type semiconductor: electrons are majority carriers, holes are minority carriers. After doping with <u>donor</u> atoms $n_o >> (n_i$ or $p_o)$, where n_o is equilibrium room temperature electron concentration.
- P-type semiconductor: holes are majority carriers, electrons are minority carriers. After doping with <u>acceptor</u> atoms $p_o >> (n_i$ or $n_o)$, where p_o is equilibrium room temperature hole concentration.

Fermi Distribution in a Doped Semiconductor


Majority carriers are electrons

$$E_{FC} \approx E_i + KT\ ln\left(\frac{n}{n_i}\right)$$


Majority carriers are holes

N-type semiconductor:

E_F is shifted towards E_c

P-type semiconductor:

E_F is shifted towards E_v

Equilibrium Carrier Distributions

- Calculating the distributions of electrons and holes in conduction and valence bands require the knowledge of the Fermi-Dirac distribution function and the density of available energy states.
- Can express this mathematically as:

$f(E) =$ Occupancy probability of electrons in conduction band.

$1 - f(E) =$ Occupancy probability of holes in valence band.

- Integrating the occupancy probability over the number of available states to determine total carrier concentration.

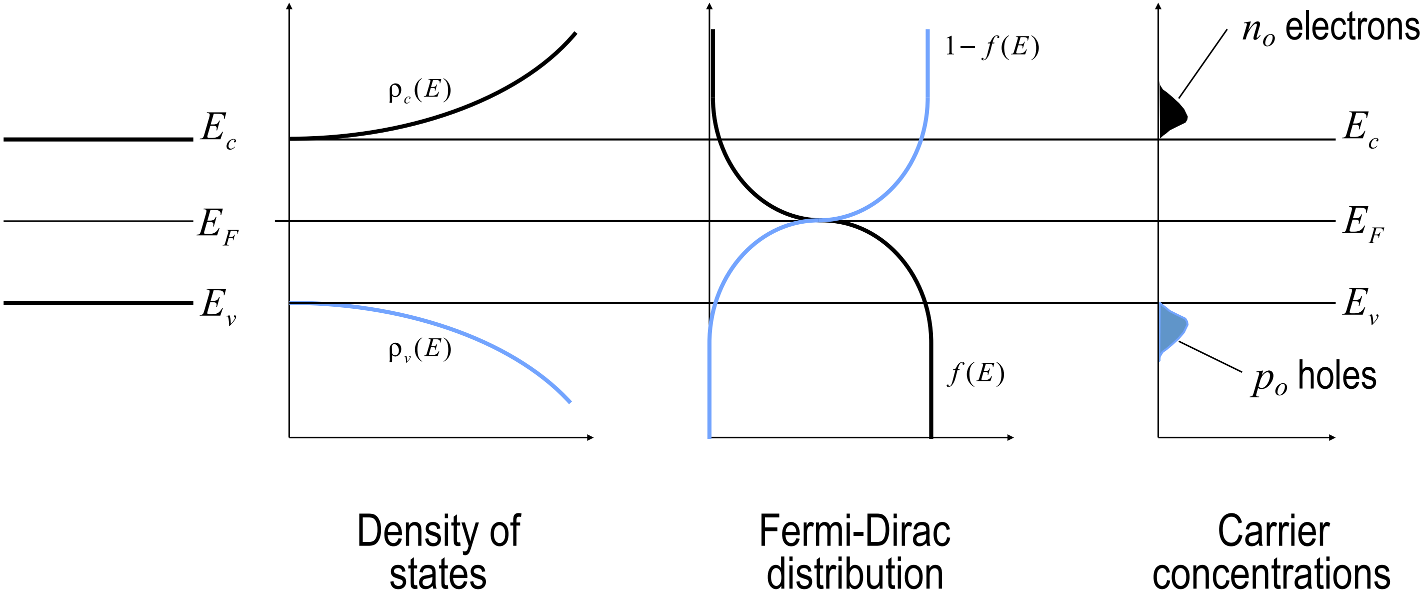
$$n_o = \int_{E_c}^{E_{top}} \rho_c(E) f(E)\, dE \qquad p_o = \int_{E_{bottom}}^{E_v} \rho_v(E)\left[1 - f(E)\right] dE$$

subscript denotes equilibrium conditions

Total number of electrons in conduction band (# / cm3)

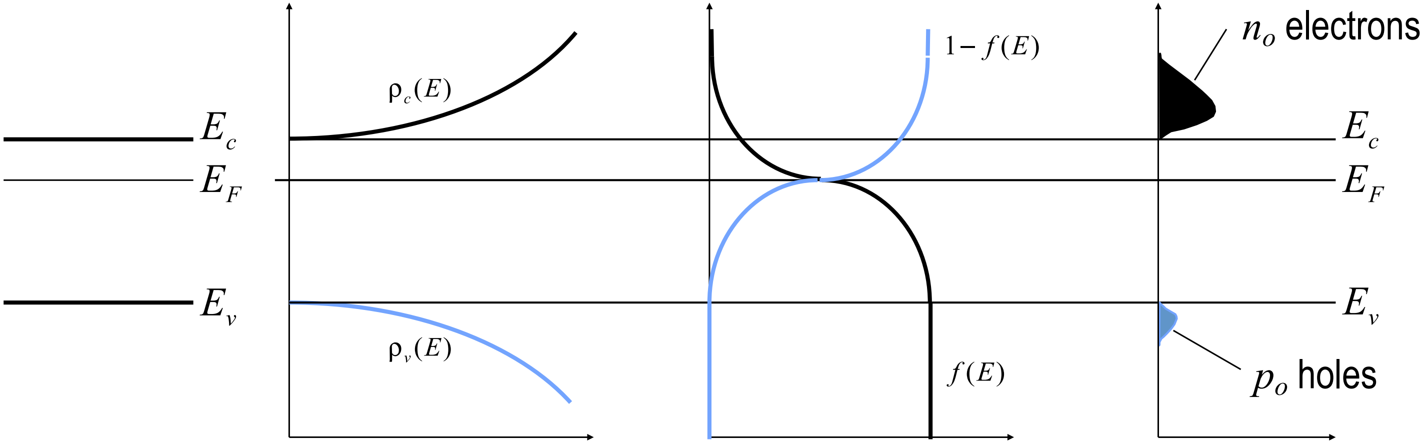
Total number of holes in valence band (# / cm3)

Carrier Distribution: Intrinsic Semiconductor



In an intrinsic material: E_F is in the middle of the band gap and $n_o = p_o$.

Carrier Distribution: N-doped Semiconductor



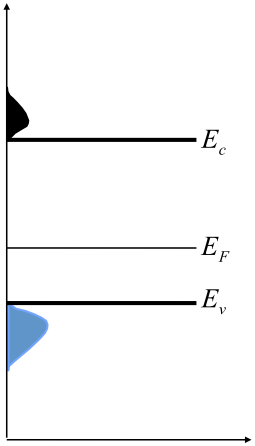
Density of states

Fermi-Dirac distribution

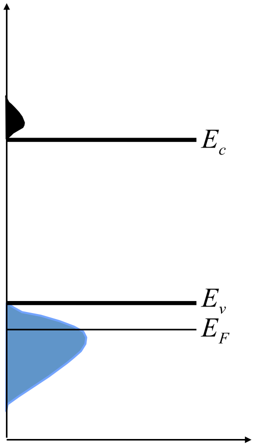
Carrier distributions

In a n-doped material: E_F is shifted towards E_c and n_o >> p_o.

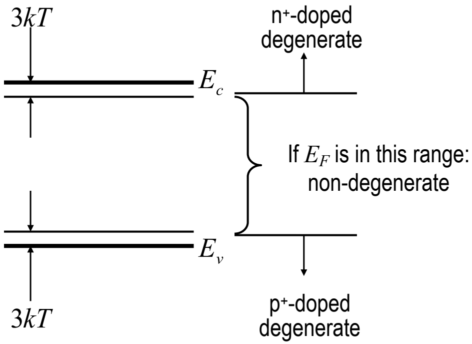
Degeneracy versus Non-Degeneracy



Non-degenerate
p-doped



Degenerate
p+-doped



n+-doped
degenerate

If E_F is in this range:
non-degenerate

p+-doped
degenerate

$$p_o = \int_{E_{bottom}}^{E_v} \rho_v(E)\left[1 - \frac{1}{e^{(E-E_F)/kT}+1}\right] dE$$

… if E_F decreases, then p_o increases

Effects of Heavy Doping

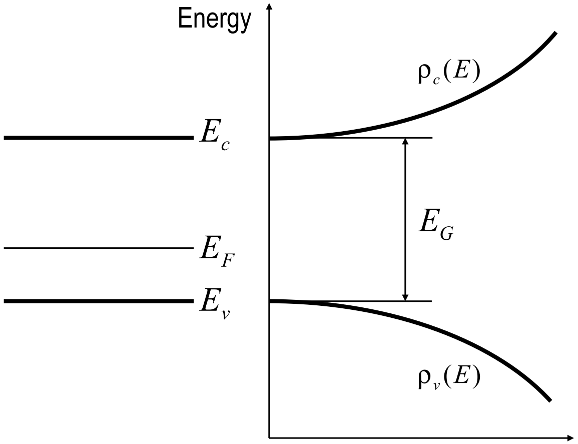
- In semiconductor lasers, the active region usually has a high density of carriers (due to high doping and current injection).

- As the doping is increased, three effects occur:
  - the quasi-Fermi levels move into the conduction and valence bands.
  - impurity atoms start interacting with each other with their discrete energy levels broadening into bands that start to merge at the band edges.
  - modifies density of states function with new states = “bandtail states”.

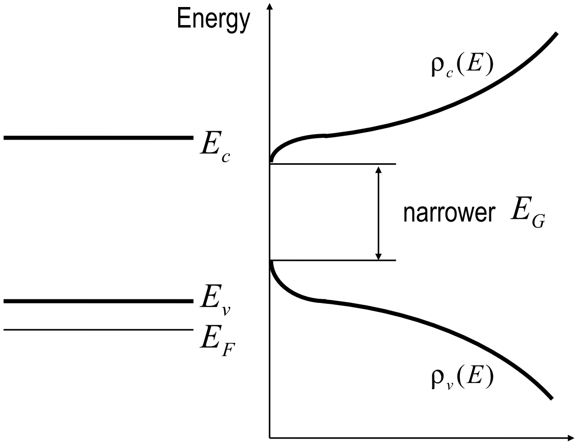
- Bandtailing reduces bandgap energy by small amount (<0.1eV).

- Bandwidth of emission spectrum increases. Hence, a semiconductor is capable of absorbing and emitting wavelengths slightly longer than the bandgap wavelength $\lambda = hc/E_G$.

Band-tailing Modifies ρ(E)



p-doped
non-degenerate



p^+-doped
degenerate

Bandgap narrowing in GaAs: $E_G = 1.424 - 1.6\text{x}10^{-8}\,(p^{1/3} + n^{1/3})$ [in eV]
(F. Stern, Journal of Applied Physics, vol. 47, p.631, 1976)

Quasi-Fermi Levels

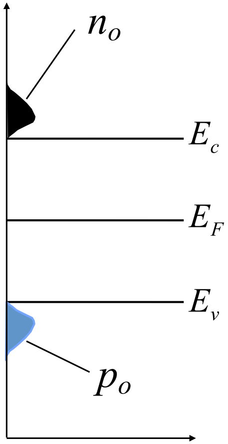
- Under equilibrium conditions, there is a direct correspondence between the Fermi level E_F and the equilibrium carrier concentrations, n_o and p_o.
- Under conditions where the thermal equilibrium is disturbed (forward-biased p-n junction or photoexcitation) the Fermi level E_F cannot be used.
- Quasi-Fermi levels are conceptual constructs used to specify the carrier concentrations inside a semiconductor in non-equilibrium conditions. This is accomplished by introducing two energies, E_{Fc}, the quasi-Fermi level for electrons, and E_{Fv}, the quasi-Fermi level for holes.
- By definition, quasi-fermi levels are related to the nonequilibrium carrier concentrations in the same way E_F is related to the equilibrium carrier concentrations:

$$n = \int_{E_c}^{E_{top}} \rho_c(E) f_c(E)\, dE = \int_{E_c}^{E_{top}} \rho_c(E) \frac{1}{e^{(E-E_{Fc})/kT}+1} dE$$

$$p = \int_{E_{bottom}}^{E_v} \rho_v(E)\left[1 - f_v(E)\right] dE = \int_{E_c}^{E_{top}} \rho_v(E)\left[1 - \frac{1}{e^{(E-E_{Fv})/kT}+1}\right] dE$$

Quasi-Fermi Levels

$$E_{FC} \approx E_i + KT\ ln\left(\frac{n}{n_i}\right)$$

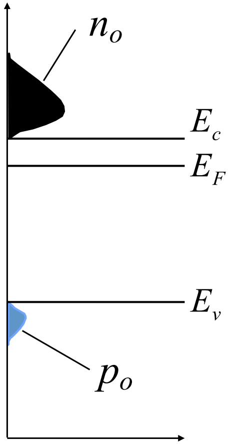


Intrinsic semiconductor
Equilibrium conditions
Electron and hole populations
described by a single Fermi level

$$n_o = \int_{E_c}^{E_{top}} \rho_c(E) f(E)\, dE$$

$$p_o = \int_{E_{bottom}}^{E_v} \rho_v(E)[1 - f(E)]\, dE$$

$$n_o = p_o = n_i \quad and \quad n_o \times p_o = n_i^2$$

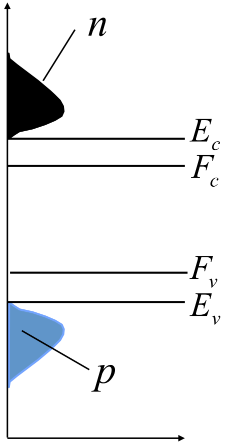


N-doped semiconductor
Equilibrium conditions
Electron and hole populations
described by a single Fermi level

$$n_o = \int_{E_c}^{E_{top}} \rho_c(E) f(E)\, dE$$

$$p_o = \int_{E_{bottom}}^{E_v} \rho_v(E)[1 - f(E)]\, dE$$

$$n_o >> p_o \quad and \quad n_o \times p_o = n_i^2$$



Photoexcited semiconductor
Steady-state conditions
Electron and hole populations
described by two (independent)
Quasi-Fermi levels

$$n = \int_{E_c}^{E_{top}} \rho_c(E) f_c(E)\, dE$$

$$p = \int_{E_{bottom}}^{E_v} \rho_v(E)[1 - f_v(E)]\, dE$$

$$n \times p \neq n_i^2$$

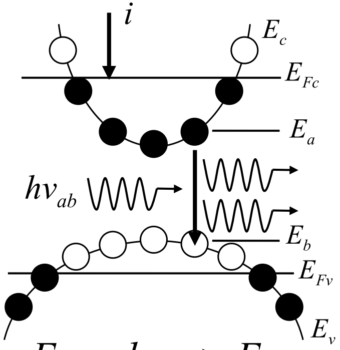
Question

A direct bandgap semiconductor has:
a band-gap of $E_g = 1eV$
$m_c = m_v/4$

- 1. Sketch the E-k diagram.
- 2. Assuming that the injected electron and hole carrier concentrations are equal, sketch the location of the quasi-Fermi energy levels, when the semiconductor is optically transparent (for a weak incident light of energy E_g).

Amplification in a Semiconductor Medium

- In common with all lasers, gain results in the active region if the rate of stimulated emission exceed that of absorption.



$$E_{ab} = h\nu_{ab} > E_G$$

⇒ Stimulated emission probability $\propto f_c(E_a)\,[1 - f_v(E_b)]$

⇒ Stimulated absorption probability $\propto f_v(E_b)\,[1 - f_c(E_a)]$

⇒ Gain coefficient

$$\propto \underbrace{f_c(E_a)[1 - f_v(E_b)] - f_v(E_b)[1 - f_c(E_a)]}_{f_c(E_a) - f_v(E_b)}$$

⇒ Gain coefficient is positive if $f_c(E_a) - f_v(E_b) > 0 \;\Rightarrow\; f_c(E_a) > f_v(E_b)$

similar to "$N_2 > N_1$"

$$\frac{1}{e^{(E_a - E_{Fc})/kT} + 1} > \frac{1}{e^{(E_b - E_{Fv})/kT} + 1} \;\Rightarrow\; E_a - E_{Fc} < E_b - E_{Fv} \;\Rightarrow\; E_{ab} = h\nu_{ab} < E_{Fc} - E_{Fv}$$

Independently derived by Basov
and Bernard + Duraffourg in 1961

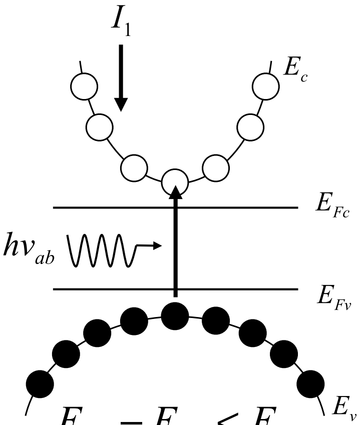
$$\boxed{E_G < h\nu_{ab} < E_{Fc} - E_{Fv}}$$

All frequencies ν_{ab} within
this range will be amplified

Absorption, Transparency and Amplification

Carriers in the conduction band: $n = \int_{E_c}^{E_{top}} \rho_c(E) f_c(E) dE$

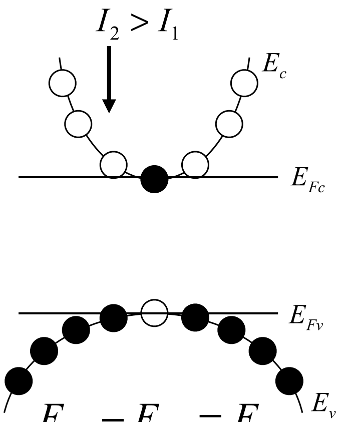
Increasing current results in an increase of E_{Fc}



$$E_{Fc} - E_{Fv} < E_G$$

Below inversion
(absorbing condition)

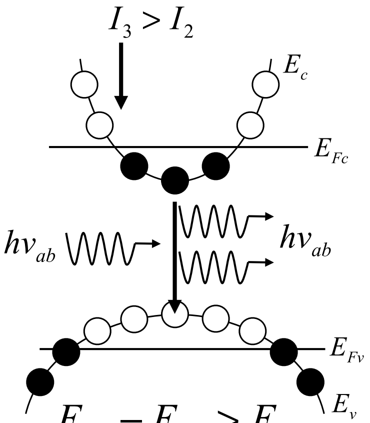
$$h\nu_{av} < E_G \Rightarrow transparent$$
$$h\nu_{av} \geq E_G \Rightarrow absorbing$$



$$E_{Fc} - E_{Fv} = E_G$$

Onset of inversion
(transparency condition)

$$h\nu_{av} \leq E_G \Rightarrow transparent$$
$$h\nu_{av} > E_G \Rightarrow absorbing$$



$$E_{Fc} - E_{Fv} > E_G$$

"Inverted" semiconductor
(amplification condition)

$$h\nu_{av} < E_G \Rightarrow transparent$$
$$E_G \leq h\nu_{av} \leq E_{Fc} - E_{Fv} \Rightarrow amplifying$$
$$h\nu_{av} > E_{Fc} - E_{Fv} \Rightarrow absorbing$$

Semiconductor Gain Coefficient

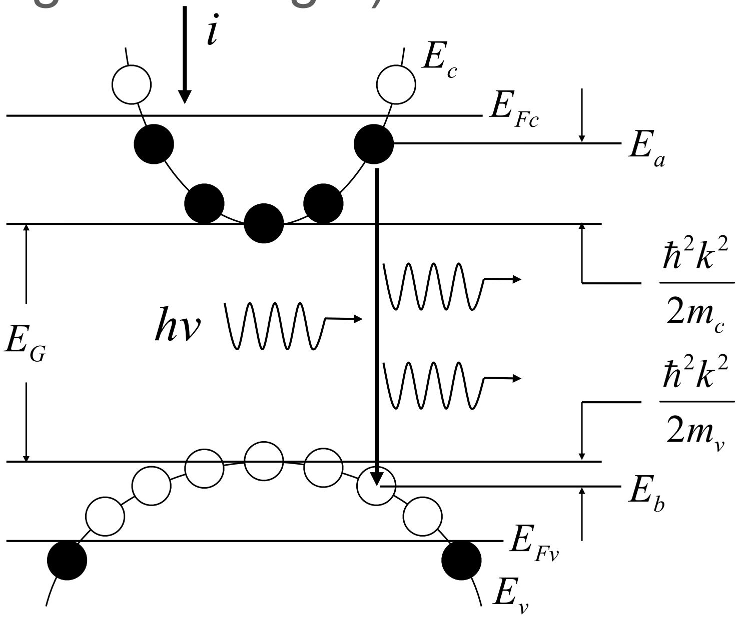
- The gain coefficient is the product of the joint density of states and the difference of the quasi-Fermi distributions, at the wavelength of interest:

$$\gamma(\nu) \propto \rho(\nu) \cdot [f_c(E_a) - f_v(E_b)]$$

$$\gamma(\nu) \propto \left(\frac{2m_r}{\hbar}\right)^{3/2} (h\nu - E_G)^{1/2} \cdot [f_c(E_a) - f_v(E_b)]$$

Optical transitions, energy levels

- How do we find E_a and E_b for a particular transition energy (lasing wavelength)?



$$h\nu = E_a - E_b = E_G + \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2 k^2}{2m_v}$$

Analysing Expression for Optical Gain

$$\gamma(\nu) = \frac{c^2}{8\pi^2 n^2 \nu_o^2 \tau}\left(\frac{2m_c m_v}{\hbar(m_c+m_v)}\right)^{3/2}\underbrace{\left(2\pi\nu_o - \frac{E_G}{\hbar}\right)^{1/2}}\;\underbrace{[f_c(E_a) - f_v(E_b)]}$$

Gain coefficient is positive if ... $2\pi\,\nu_o > \frac{E_G}{\hbar}$ $\quad f_c(E_a) > f_v(E_b)$

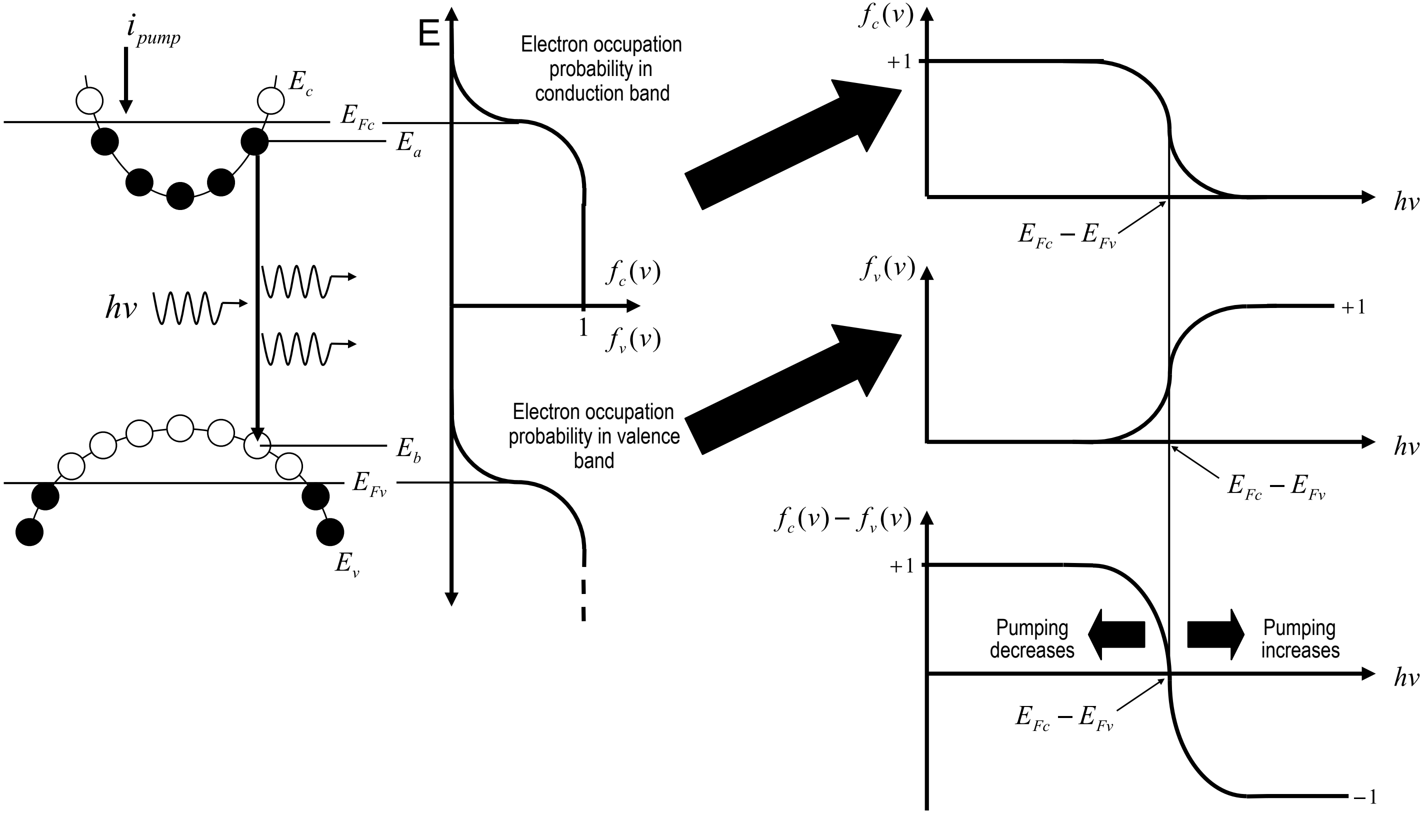
$$h\nu_o > E_G \qquad h\nu_o = E_a - E_b < E_{Fc} - E_{Fv}$$

Only frequencies whose photon energies fall within this range are amplified

$$\boxed{E_G < h\nu_o < E_{Fc} - E_{Fv}}$$

Constructing a Plot of $f_c(v)$ - $f_v(v)$

(subject to the constraint: $hv = E_a - E_b$)



Electron occupation probability in conduction band

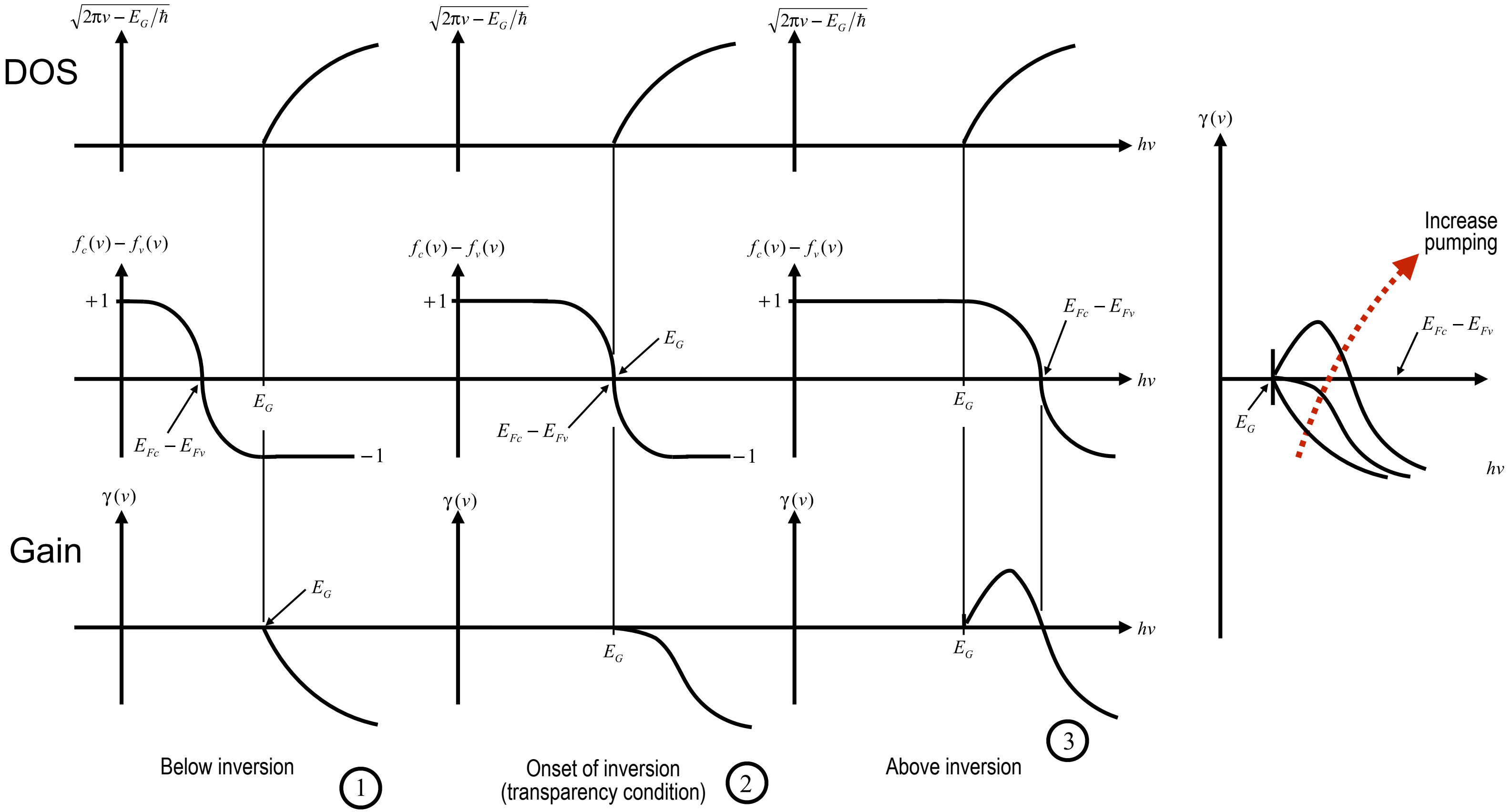
Electron occupation probability in valence band

Pumping decreases

Pumping increases

Constructing a gain plot:

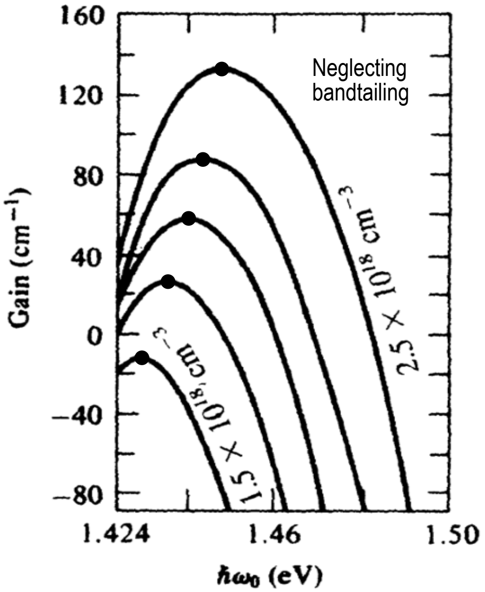
$$\gamma(\nu) = \frac{c^2}{8\pi^2 n^2 \nu_o^2 \tau}\left(\frac{2m_c m_v}{\hbar(m_c+m_v)}\right)^{3/2}\left(2\pi\nu_o - \frac{E_G}{\hbar}\right)^{1/2}\left[f_c(E_a)-f_v(E_b)\right]$$



Calculated Gain Coefficient for GaAs

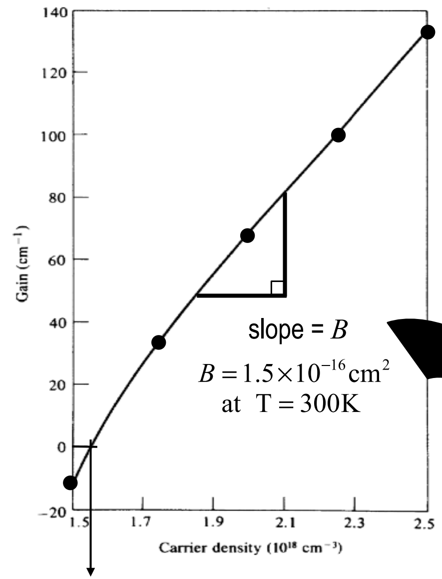
(Yariv figures 15-7 and 15-8)

Gain vs photon energy
(carrier density as a parameter)



Neglecting
bandtailing

Peak gain vs carrier density



slope = B

$B = 1.5\times10^{-16}\,\mathrm{cm}^2$
at T = 300K

$N_{tr} = 1.55\times10^{18}\, cm^{-3}$

m_c = (0.067)m$_o$

m_v = (0.48)m$_o$

T_2 = 0.5 ps

τ = 3 ns

E_G = 1.424 eV

T = 300 K

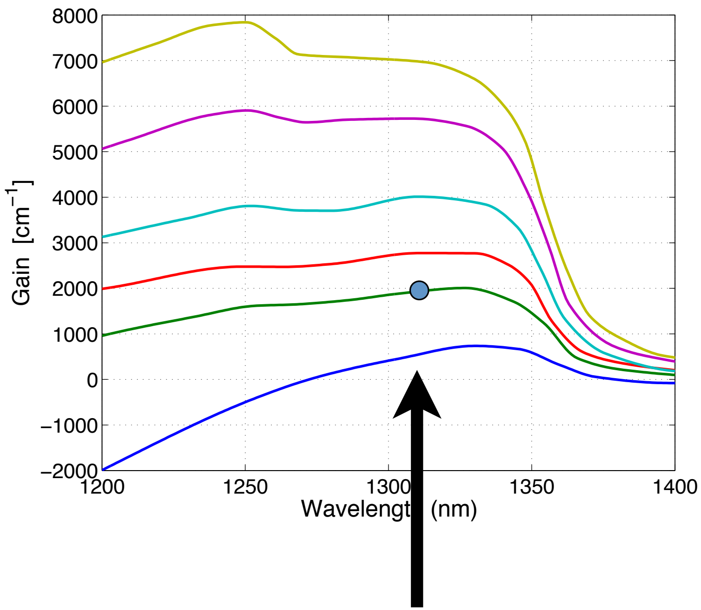
$$\boxed{\gamma_{\max} = B(N - N_{tr})}$$

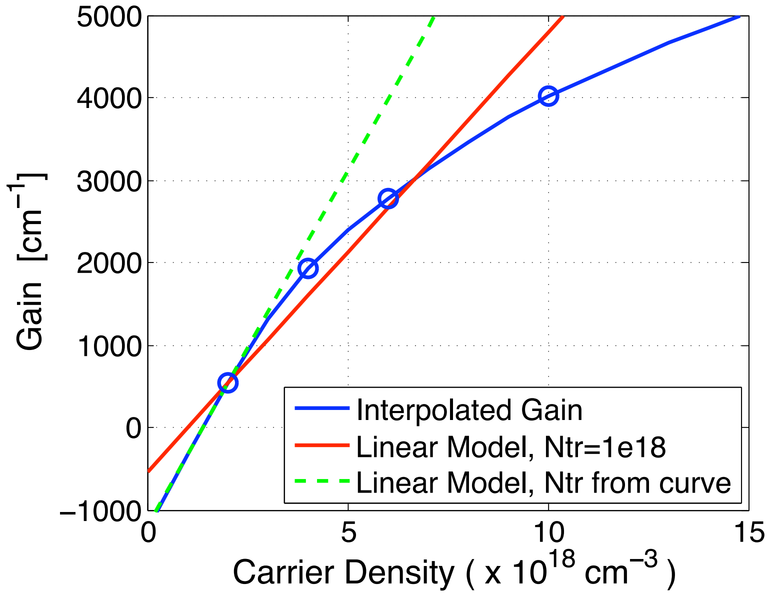
N_{tr} is the carrier density required to achieve transparency (onset of inversion).

Optical gain model

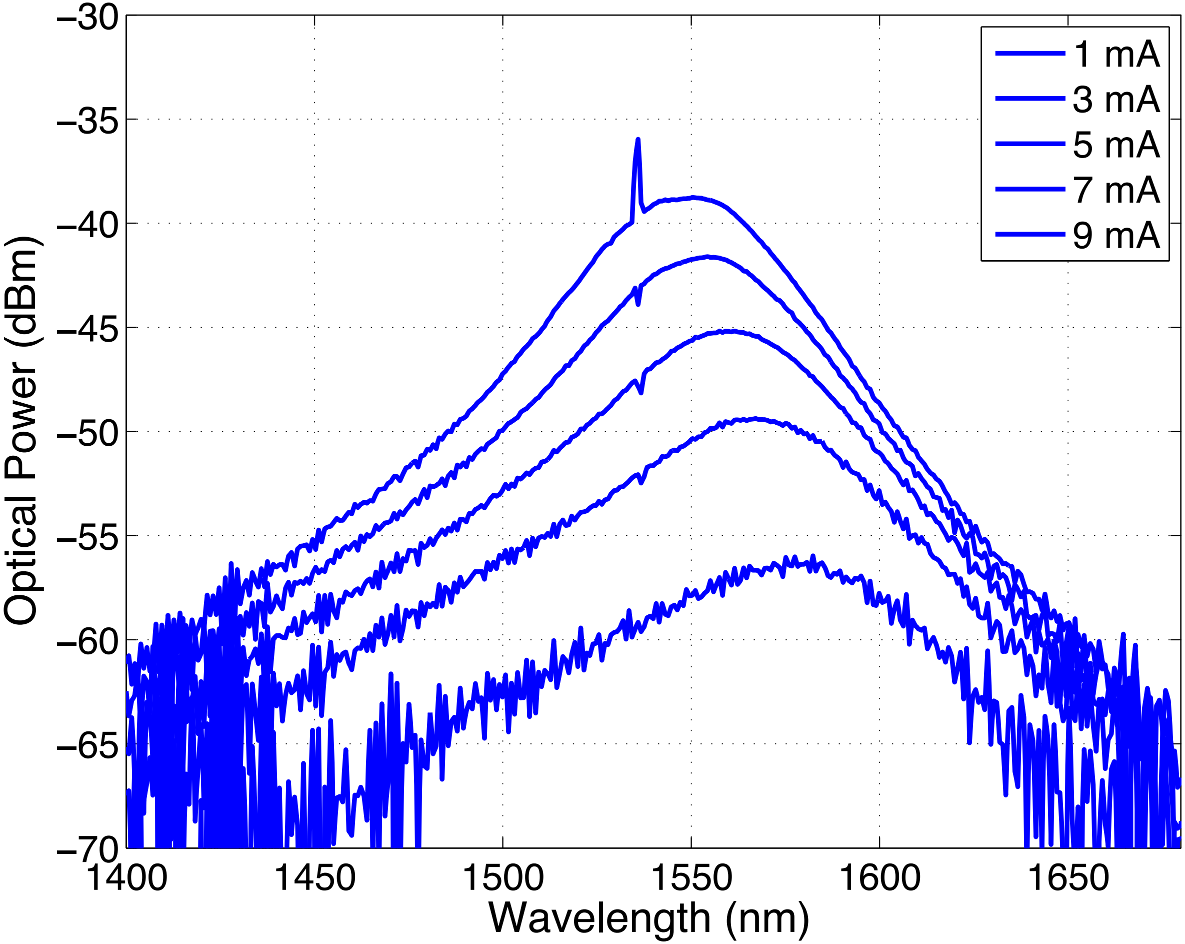
- Linear gain model

estimate the gain using linear function, with $\gamma = B(N - N_{tr})$.





Optical Spectrum – Spontaneous Emission

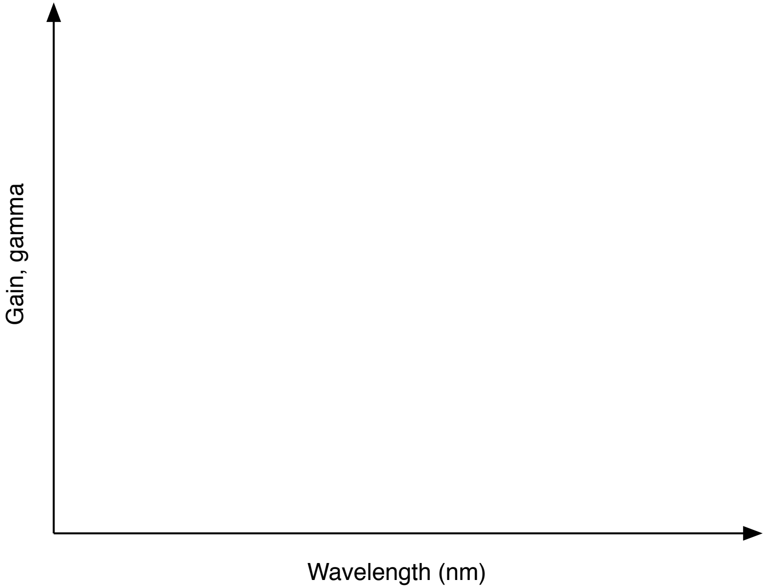


Question

a) Evaluate and plot the gain $\gamma(\omega)$ of an inverted GaAs crystal under the following conditions:

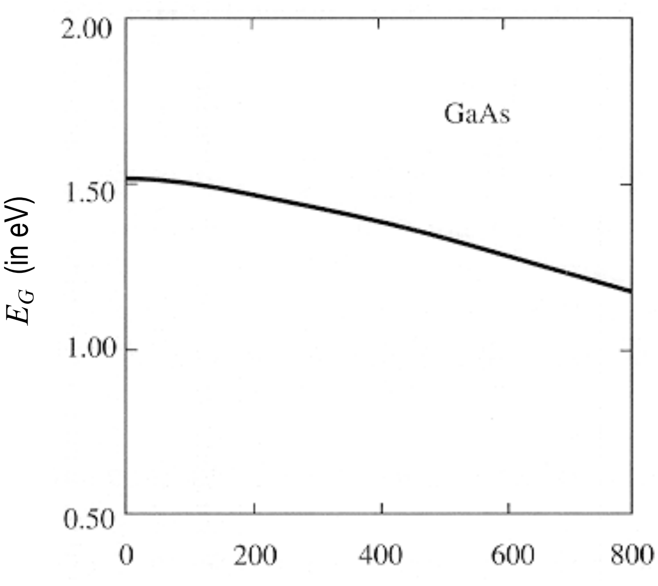
$$N_{elec} = N_{hole} = 3 \times 10^{18} cm^{-3}$$
$$m_c = 0.07 m_{electron}$$
$$m_h = 0.4 m_{electron}$$
$$T = 0K$$
$$E_g = 1.45 eV$$
$$T_2 = \infty$$

b) How would the gain profile change if the temperature is raised? (estimate and sketch)


Gain, gamma
Wavelength (nm)

Temperature Dependence of E_G

Temperature Dependence of E_G for GaAs



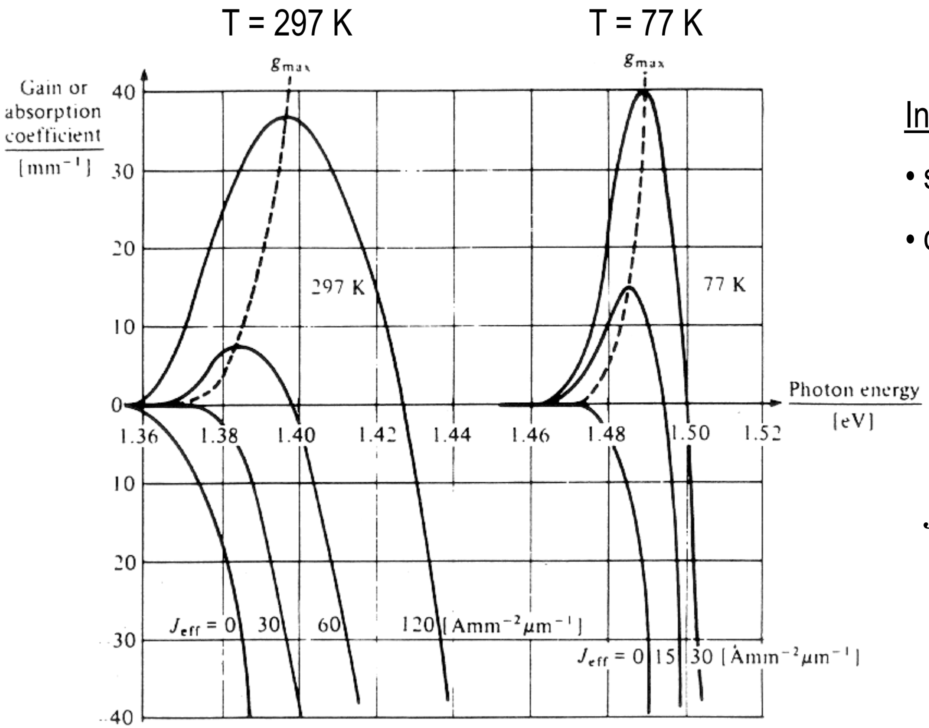
Temperature (K)

- As the temperature of a semiconductor is varied, the lattice contracts or expands, electron-lattice interaction change and impurity potentials are also affected.
- The macroscopic effect of all these changes obeys the following empirical relation:

$$E_G(T) = E_G(0) - \frac{AT^2}{T+B}$$

	E_G (0K) (eV)	A (eV/K^2)	B (K)
GaAs	1.519	5.405x10^{-4}	204
InP	1.421	4.906x10^{-4}	327

Temperature Dependence of Gain Spectrum



Calculated gain coefficient of p-type GaAs as a function of photon energy with effective pumping current as a parameter

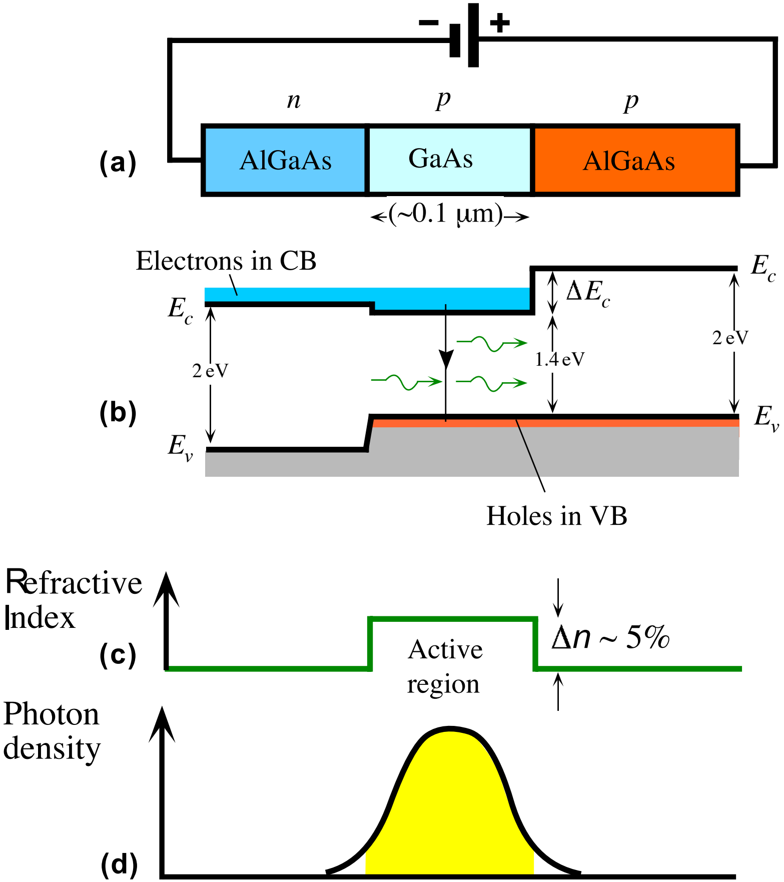
Increase in temperature results in:

- shift of gain curve to lower energy.
- decrease in value of γ_{max}.
  - wider fermi distribution

$$J = \text{current density } [\text{A/cm}^2]$$

$$I = J \times area$$

Carrier & Light Confinement



(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs).

(b) Simplified energy band diagram under a large forward bias. Lasing recombination takes place in the *p*-GaAs layer, the *active layer*

(c) Higher bandgap materials have a lower refractive index

(d) AlGaAs layers provide lateral optical confinement.

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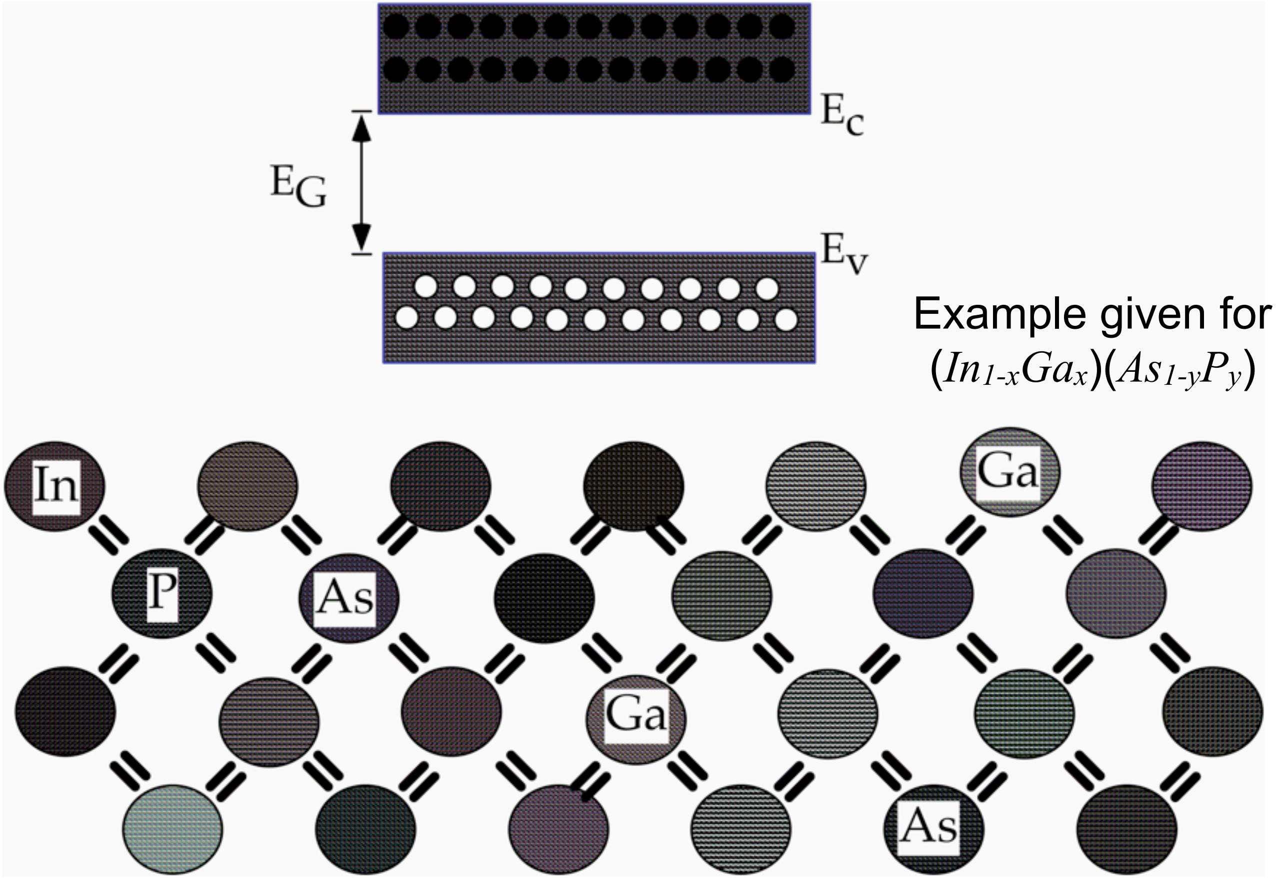
Alloying to Alter Band Gap Energy

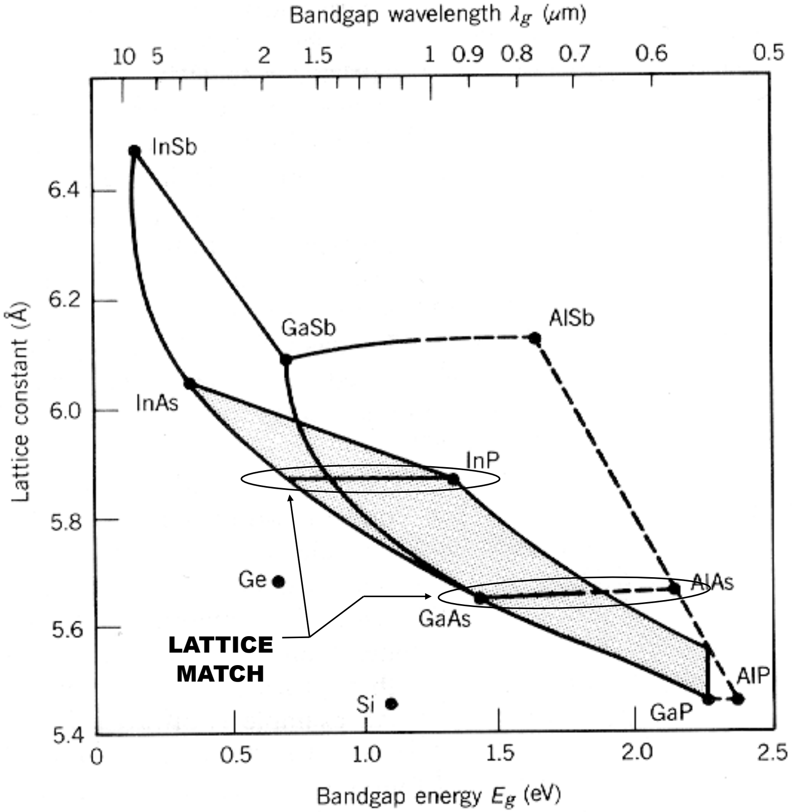
- The bandgap energy of III-V direct gap binary semiconductors limit the operating wavelength of semiconductor lasers:
  - GaAs $\Rightarrow E_G = hc/\lambda$ = 1.42 eV $\Rightarrow$ corresponds to λ = 870 nm.
  - InP $\Rightarrow E_G = hc/\lambda$ = 1.35 eV $\Rightarrow$ corresponds to λ = 919 nm.
- Can alloy semiconductor to alter gap energy assuming can grow high purity material.
- Epitaxial growth of lattice matched semiconductors on inexpensive, low defect density substrates is required. GaAs and InP are only common substrates.
- Can grow binary, ternary, and quaternary alloys.
- Common systems include GaAs/AlGaAs for 0.7-0.9 μm emission, InGaAsP/InP for 0.9-1.65 μm emission.

Alloying to Modify EG



Example given for $(In_{1-x}Ga_x)(As_{1-y}P_y)$

Lattice Constant, Bandgap Energies and Wavelengths



A ternary compound can be formed by motion along the lines joining the two binary compounds.

A quaternary compound is represented by a point in the area formed by its four binary components.

$(In_{1-x}Ga_x)(As_{1-y}P_y)$ = shaded area.

Solid curve = direct gap material

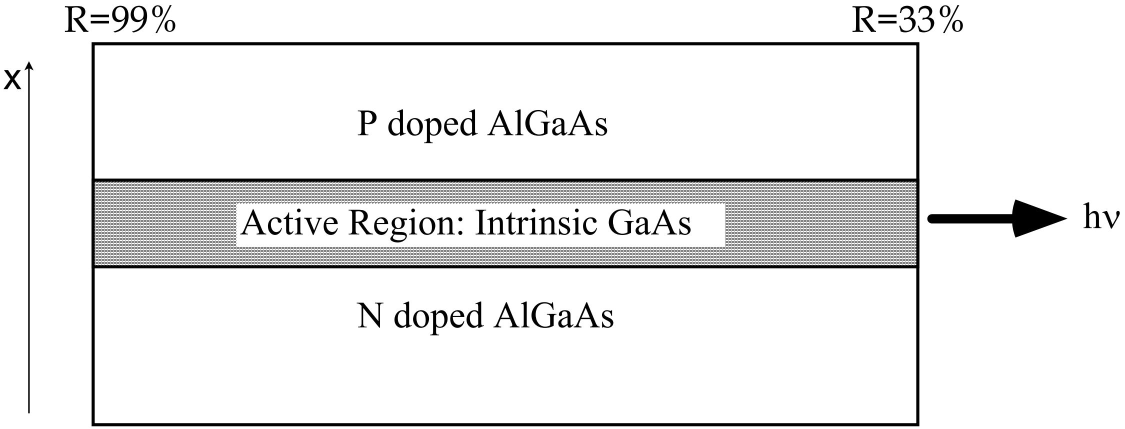
Dashed Curve = indirect gap material

A material may have a direct bandgap for one mixing ratio and indirect gap for a different ratio.

$Al_xGa_{1-x}As$ with $x < 0.37 \Rightarrow$ direct gap

$Al_xGa_{1-x}As$ with $x > 0.37 \Rightarrow$ indirect gap

Question – Heterostructure



R=99%

R=33%

x

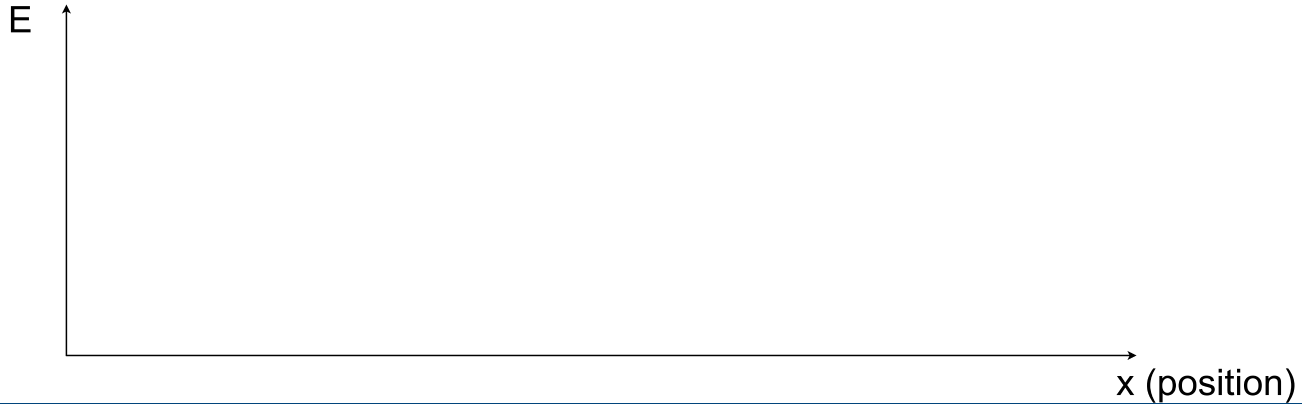
P doped AlGaAs

Active Region: Intrinsic GaAs

N doped AlGaAs

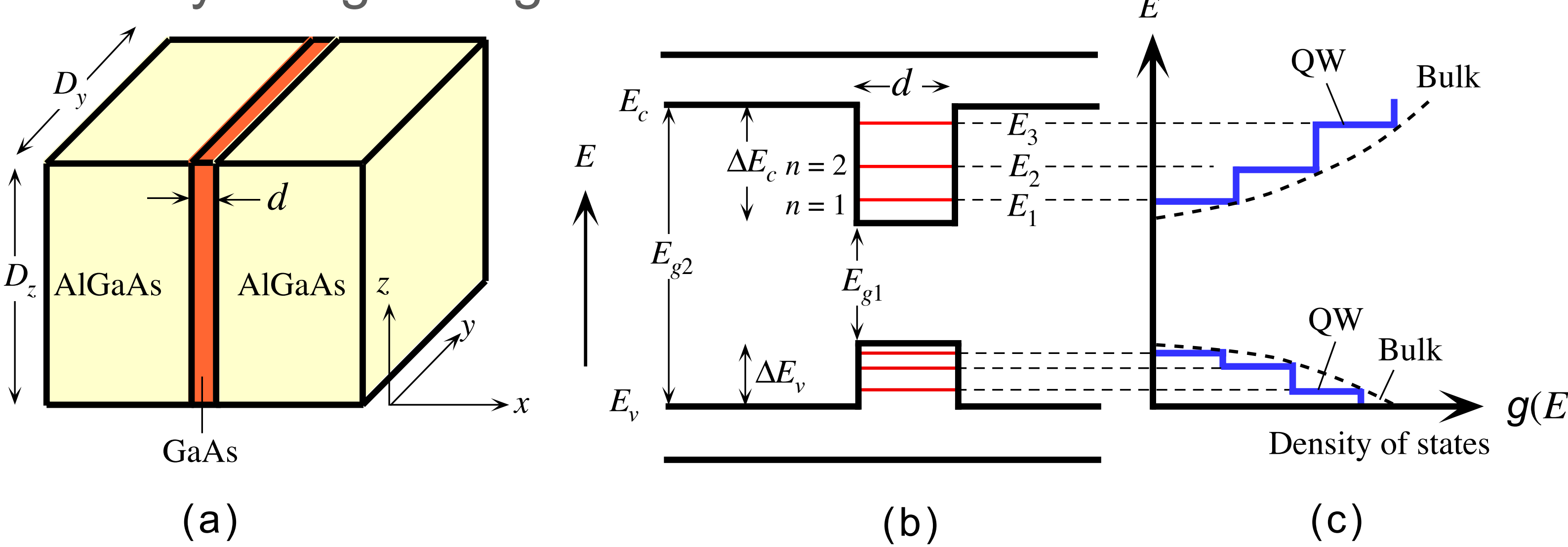
hν

a) [3 pts.] Draw the equilibrium band diagram of the laser indicating the location of the following energy levels: E_I, E_F, E_C, and E_V.



Quantum Wells

- Very thin gain region



A quantum well (QW) device. (a) Schematic illustration of a quantum well (QW) structure in which a thin layer of GaAs is sandwiched between two wider bandgap semiconductors (AlGaAs). (b) The conduction electrons in the GaAs layer are confined (by ΔE_c) in the x-direction to a small length d so that their energy is quantized. (c) The density of states of a two-dimensional QW. The density of states is constant at each quantized energy level.

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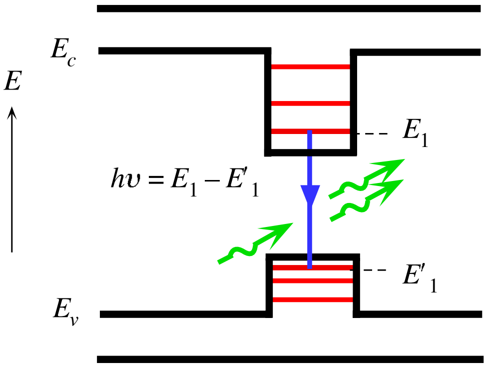
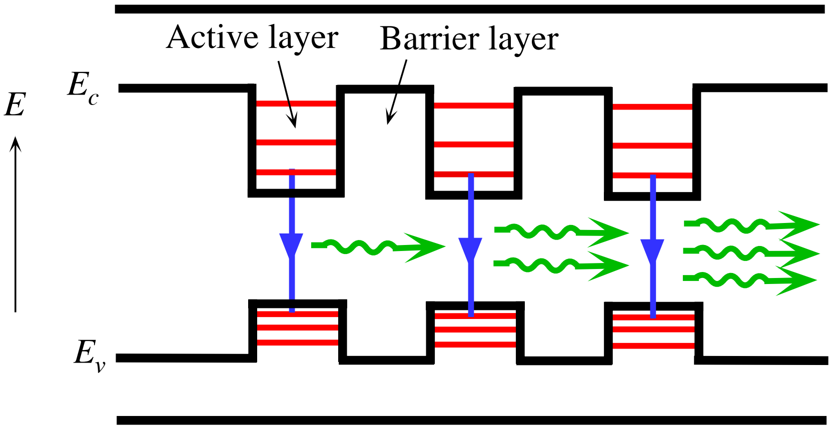


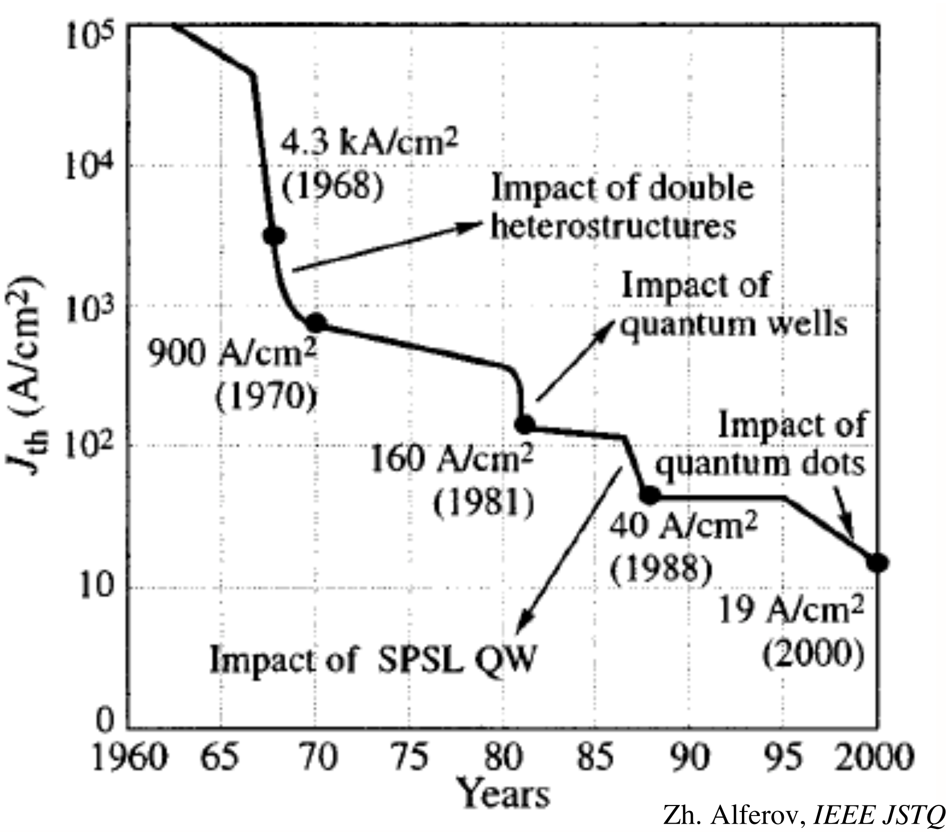
In single quantum well (SQW) lasers electrons are injected by the forward current into the thin GaAs layer which serves as the active layer. Population inversion between E_1 and E'_1 is reached even with a small forward current which results in stimulated emissions.



A multiple quantum well (MQW) structure. Electrons are injected by the forward current into active layers which are quantum wells.

- Transparency current (i.e. onset of inversion) – which has highest? which has lowest?
  - pn homo-junction, hetero-junction, QW structure

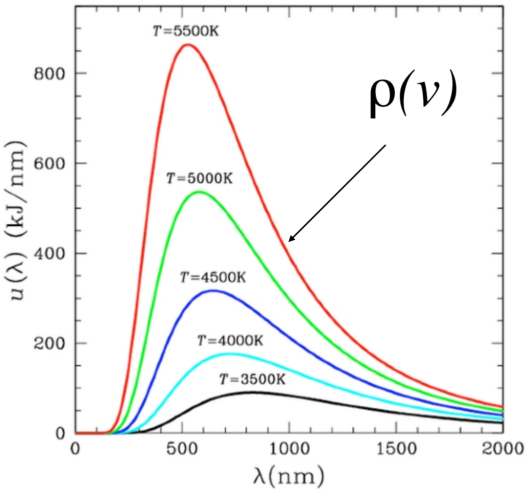
Impact of junction technology on laser threshold (current density J_{th})



Zh. Alferov, *IEEE JSTQE* **6** 832 (2000)

Black-Body Radiation

- A black-body absorbs with 100% efficiency all the radiation falling on it, irrespective of the radiation frequency (a close approximation of a black-body is an enclosed cavity with a small hole).
- If the inside of the cavity is in thermal equilibrium, it must lose as much energy as it absorbs: emissions from the hole are therefore characteristic of the equilibrium temperature T in the cavity.



Planck's law

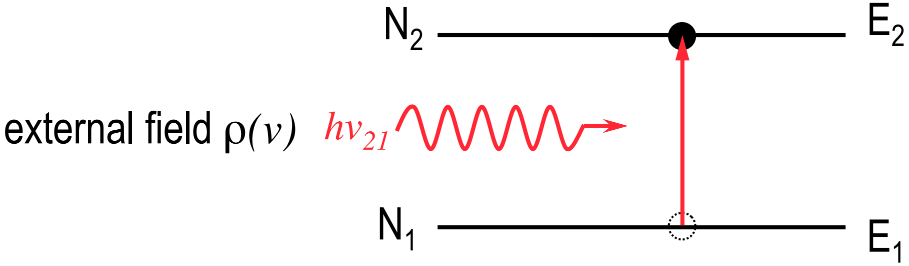
$$\rho(v) = \frac{8\pi n^3 h v^3}{c^3 (e^{hv/kT} - 1)}$$

(Energy Density)

http://en.wikipedia.org/wiki/Planck's_law_of_black_body_radiation

Process of Stimulated Absorption

- Electrons can make stimulated transitions in the upward direction between energy levels of an atom by absorbing energy from $\rho(v)$:



- The rate of change of population density N_1 is:

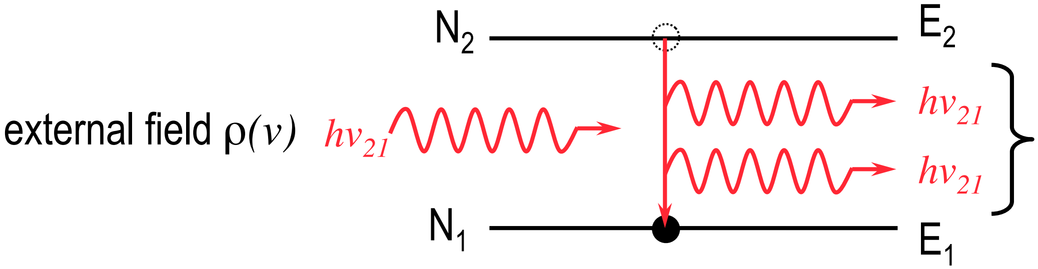
$$\frac{dN_1}{dt} = -N_1 B_{12} \int_{-\infty}^{+\infty} g(v)\rho(v)dv = -N_1 B_{12}\rho(v_{21})$$

B_{12} is called the Einstein coefficient for stimulated absorption.

$\rho(v)$ is the energy density in the frequency range close to v_{21}.

Process of Stimulated Emission

- Electron transitions can be stimulated by the action of an external radiation field.



output photons have:
- same direction
- same frequency
- same phase

- $\rho(\nu)$ = energy density of the applied radiation field at frequency ν. (energy per unit volume per unit frequency interval: J.m^{-3}.Hz^{-1}).
- The rate of change of population density N_2 is then:

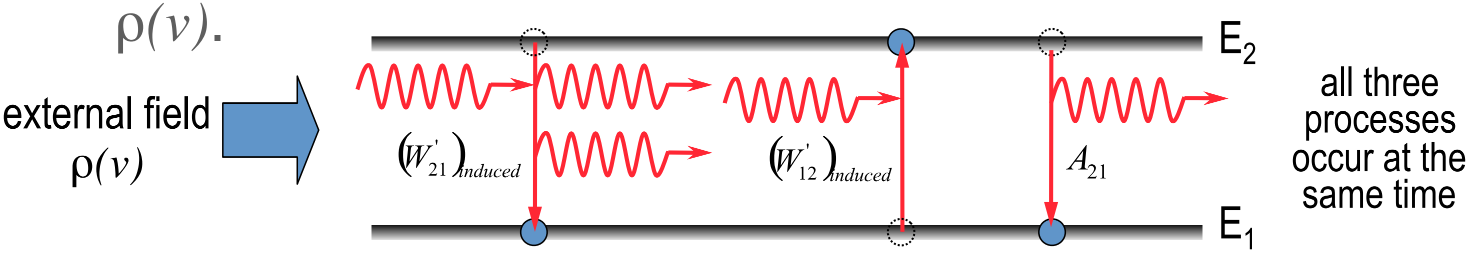
$$\frac{dN_2}{dt} = -N_2 B_{21} \int_{-\infty}^{+\infty} g(\nu)\rho(\nu)d\nu = -N_2 B_{21}\rho(\nu)$$

B_{21} is called the Einstein coefficient for stimulated emission.

$\rho(\nu)$ is the energy density in the frequency range close to ν_{21}.

Derivation of transition rates

- Consider the interaction of an assembly of identical atoms with an external **black-body** radiation field, $\rho(\nu)$.



Total rate of 2→1 transitions = $W'_{21} = (W'_{21})_{induced} + A_{21} = B_{21}\rho(\nu) + A_{21}$

Total rate of 1→2 transitions = $W'_{12} = (W'_{12})_{induced} = B_{12}\rho(\nu)$

Under **thermal equilibrium** ⇒ $N_2 W'_{21} = N_1 W'_{12}$

Number of electrons going up must equal number of electrons going down

Relation between the A and B Coefficients

$$\underbrace{\rho(\nu)=\frac{8\pi n^3 h\nu^3}{c^3\left(e^{h\nu/kT}-1\right)} \qquad \frac{N_2}{N_1}=e^{-h\nu/kT} \qquad N_2W_{21}^{'}=N_1W_{12}^{'}}_{\text{leads to:}}$$

$$B_{12}=B_{21} \qquad \frac{A_{21}}{B_{21}}=\frac{8\pi n^3 h\nu^3}{c^3}$$

Induced transition rate can then be rewritten as:

$$\left(W_{21}^{'}\right)_{induced}=\left(W_{12}^{'}\right)_{induced}=W_i^{'}=\frac{A_{21}c^3}{8\pi n^3 h\nu^3}\rho(\nu)=\frac{c^3}{8\pi n^3 h\nu^3 t_{spont}}\rho(\nu)$$

Induced Transition Rate Wi(v)

- In a laser, ρ(*v*) is monochromatic (~ delta function). ρ(*v*) will induce transitions at the frequency v_{21}.
- Only the part of ρ(*v*) which overlaps with *g*(*v*) will contribute to induced transitions.

$$W_i(v) = \int_{-\infty}^{+\infty} W_i'(v) g(v) d(v)$$

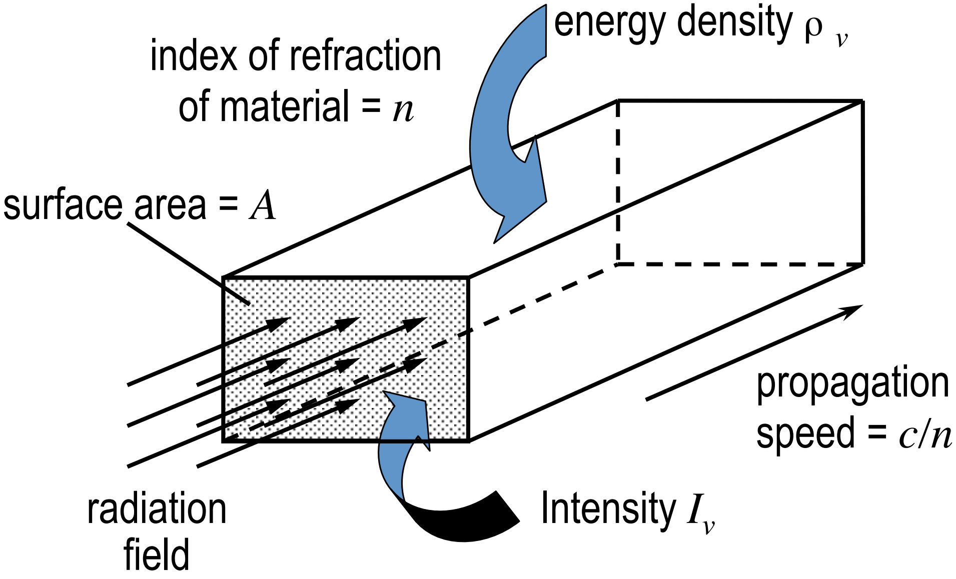
$$W_i(v) = \frac{c^3}{8\pi n^3 h t_{spont}} \int_{-\infty}^{+\infty} \frac{\rho(v) g(v)}{v^3} d(v') \cong \frac{c^3 \rho_v}{8\pi n^3 h v^3 t_{spont}} g(v)$$

- Where ρ_v is the energy density per unit frequency.

note: $\rho_\nu = U_o$ (in Yariv)

Relation between Energy Density and Intensity

- Energy density of a radiation field = ρ_ν ⇒ in J.cm^{-3}
- Intensity of radiation field = I_ν ⇒ in W.cm^{-2}



conservation of energy:

$$\rho_\nu \times A \times \frac{c}{n} = I_\nu \times A$$

$$\boxed{\rho_\nu = \frac{n}{c} I_\nu}$$

Rewriting Induced Transition Rate Wi(v)

$$W_i(v) = \frac{c^3 \rho_v}{8\pi n^3 h v^3 t_{spont}} g(v) = \frac{c^2 I_v}{8\pi n^2 h v^3 t_{spont}} g(v)$$

$$\boxed{W_i(v) = \frac{c^2 I_v}{8\pi n^2 h v^3 t_{spont}} g(v)}$$

Rate of induced transitions $W_i(v)$ is:
- directly proportional to lineshape function $g(v)$.
- directly proportional to intensity of optical field I_v.
- inversely proportional to spontaneous lifetime t_{spont}.

Absorption and Amplification

- Both absorption and amplification occur simultaneously.
- Use 2 level gain (See Yariv Ch. 5):

gain coefficient:

$$\gamma(\nu) = (N_2 - N_1)\left(\frac{c^2}{8\pi n^2 \nu^2 t_{spont}}\right) g(\nu)$$

- If $N_1 > N_2$ (as in thermal equilibrium) $\Rightarrow$ intensity is <u>attenuated.</u>
- If $N_2 > N_1$ (population is "inverted") $\Rightarrow$ intensity is <u>amplified.</u>

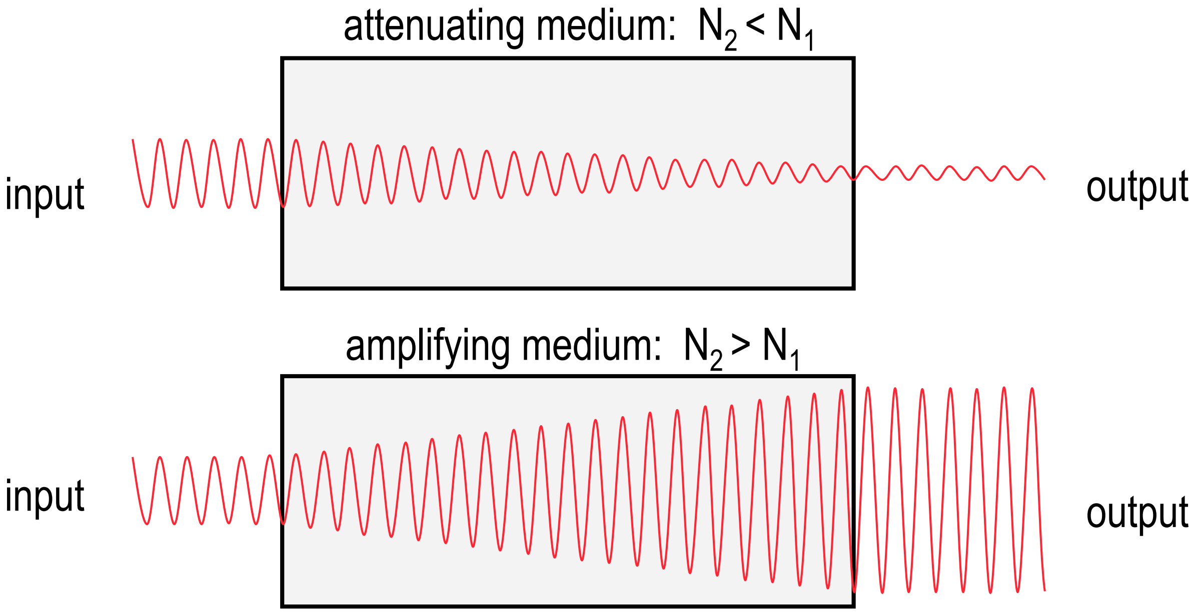
$$I_\nu(z) = I_\nu(0) e^{\gamma(\nu) z}$$

$$\frac{N_2}{N_1} = e^{-h\nu/kT}$$

➡

The "*population inversion*" condition is often referred to as one of "negative temperature" in the literature.

Attenuating vs. Amplifying Medium



attenuating medium: $N_2 < N_1$

input

output

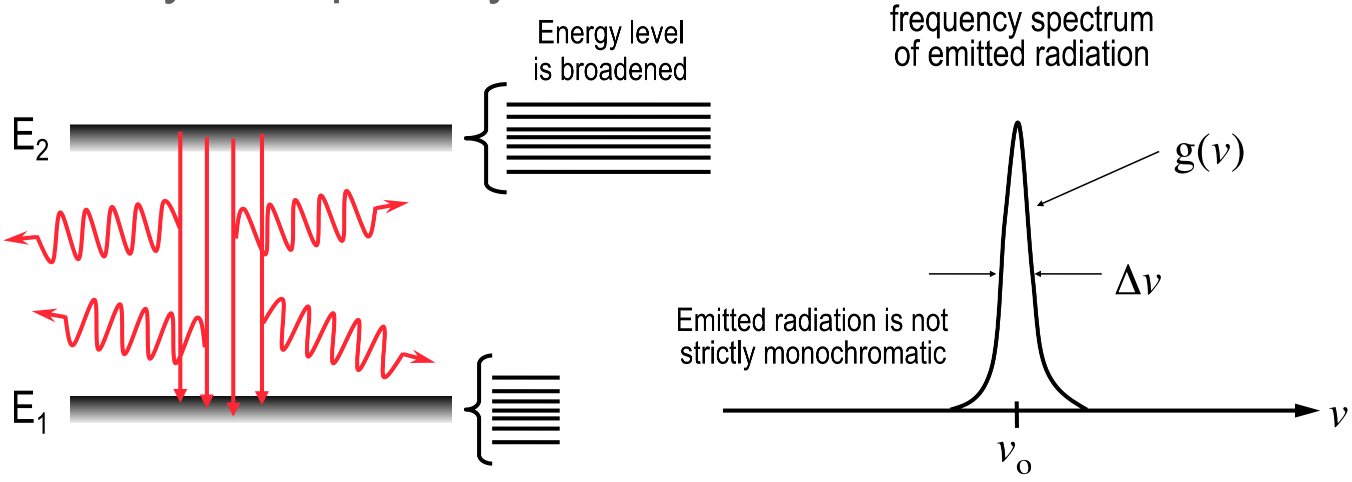
amplifying medium: $N_2 > N_1$

input

output

Concept of Lineshape Function

- When an electron changes its energy spontaneously the emitted radiation is not (as might perhaps be expected) all at the same frequency. Real energy levels are not infinitely “sharp”; they are smeared out or *broadened*.


Energy level is broadened

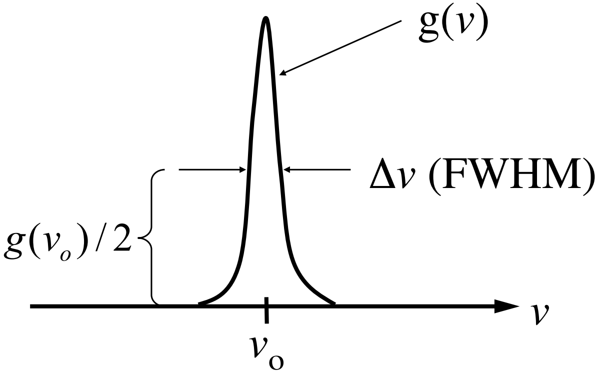
frequency spectrum of emitted radiation

Emitted radiation is not strictly monochromatic

- The function $g(\nu)$ is referred to as the *lineshape function*.

Normalized Lorentzian Lineshape Function

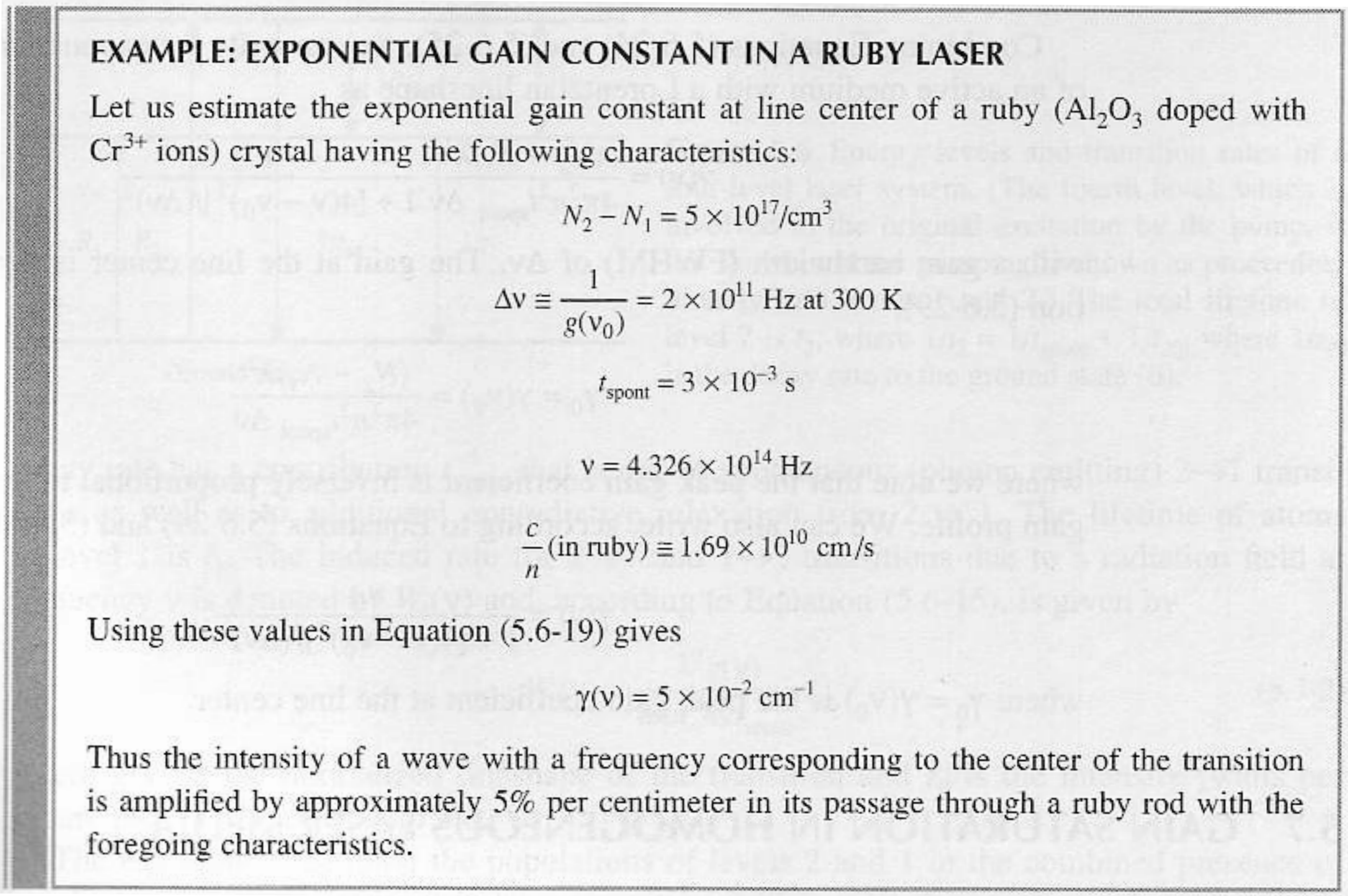
$$\boxed{g(\nu) = \frac{\Delta\nu}{2\pi[(\nu-\nu_o)^2 + (\Delta\nu/2)^2]}}$$



- Since natural broadening is the same for each atom it is said to be a ***homogeneous*** broadening mechanism.

Homogeneous and Inhomogeneous Broadening

- The lineshape function, $g(\nu)$, can arise from the following:
  - *homogeneous* mechanisms (i.e. identical):
    - **Natural lineshape** (Lorentzian) – from damping / finite lifetime
    - **Atom-phonon collisions in crystals**: phonons - acoustic energy, interact with atoms modifying phase of emission
    - **Atom-atom collisions**.
  - *inhomogeneous* mechanisms (i.e. individual, distinguishable):
    - **Crystal strain and imperfections**
    - **Doppler broadening**: velocities of particles in a gas



EXAMPLE: EXPONENTIAL GAIN CONSTANT IN A RUBY LASER

Let us estimate the exponential gain constant at line center of a ruby (Al_2O_3 doped with Cr^{3+} ions) crystal having the following characteristics:

$$N_2 - N_1 = 5 \times 10^{17}/\text{cm}^3$$

$$\Delta\nu \equiv \frac{1}{g(\nu_0)} = 2 \times 10^{11}\ \text{Hz at } 300\ \text{K}$$

$$t_{\text{spont}} = 3 \times 10^{-3}\ \text{s}$$

$$\nu = 4.326 \times 10^{14}\ \text{Hz}$$

$$\frac{c}{n}(\text{in ruby}) \equiv 1.69 \times 10^{10}\ \text{cm/s}$$

Using these values in Equation (5.6-19) gives

$$\gamma(\nu) = 5 \times 10^{-2}\ \text{cm}^{-1}$$

Thus the intensity of a wave with a frequency corresponding to the center of the transition is amplified by approximately 5% per centimeter in its passage through a ruby rod with the foregoing characteristics.

Summary of Results

rate of induced transitions (s^{-1}) →

$$W_i(v) = \frac{c^2 I_v}{8\pi n^2 h v^3 t_{spont}} g(v)$$

net power per unit volume (W.cm^{-3}) →

$$\frac{dP}{dV} = \frac{dI}{dz} = (N_2 - N_1) W_i h v$$

intensity per unit length (W.cm^{-3}) →

$$\frac{dI_v}{dz} = (N_2 - N_1) \frac{c^2 g(v)}{8\pi n^2 v^2 t_{spont}} I_v$$

intensity (W.cm^{-2}) →

$$I_v(z) = I_v(0)\, e^{\gamma(v) z}$$

neglecting saturation

gain coefficient (cm^{-1}) →

$$\gamma(v) = (N_2 - N_1) \frac{c^2}{8\pi n^2 \nu^2 t_{spont}} g(\nu)$$

Properties of Gain Coefficient

$$\gamma(v) = (N_2 - N_1)\frac{c^2}{8\pi\, n^2 \nu^2 t_{spont}} g(\nu)$$

- Gain coefficient dependence:
    - Population inversion: $N_2 - N_1$
    - Transition lineshape function: $g(v)$
    - Spontaneous lifetime of transition: t_{spont}
- How to create and sustain a population inversion?
- What happens to the population inversion as field intensity grows large?
- What limits the intensity from growing without bound?